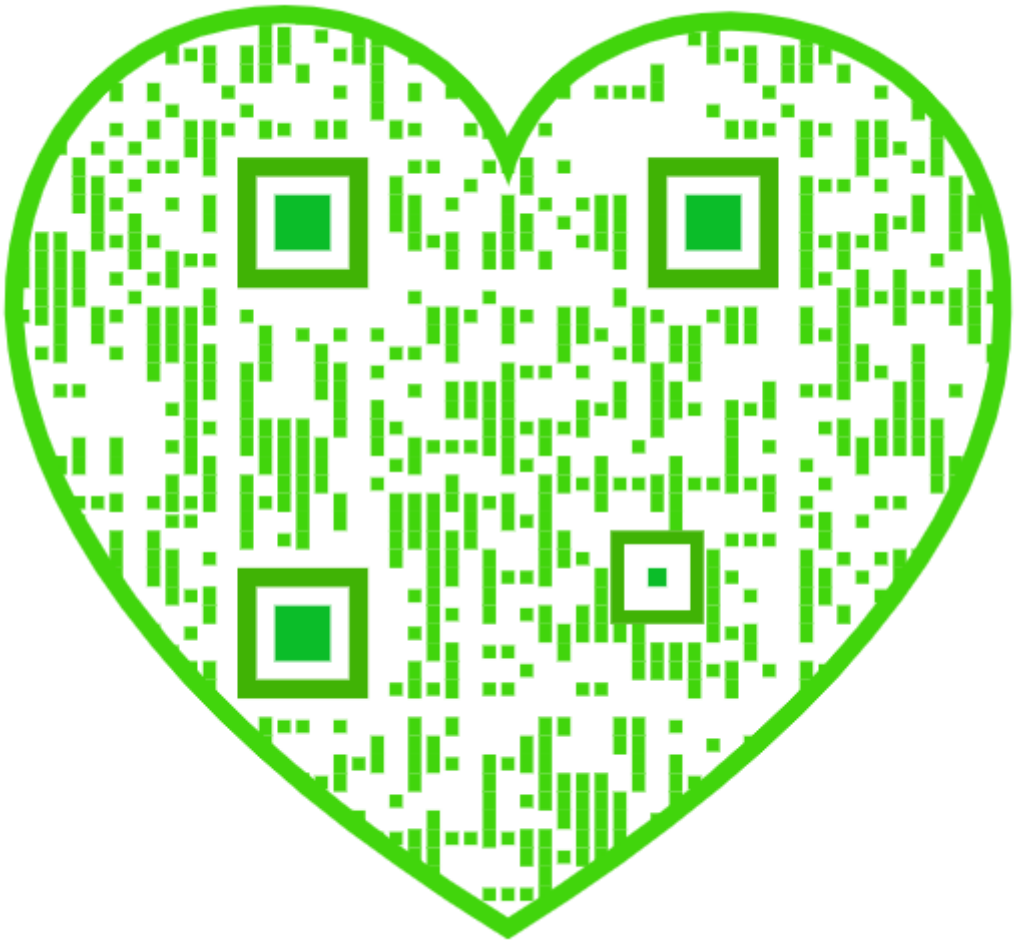

INDUSTRIAL ARITHMETIC

RORAY

INDUSTRIAL ARITHMETIC

AN ELEMENTARY TEXT FOR BOYS IN INDUSTRIAL, TECHNICAL, VOCATIONAL AND TRADES SCHOOLS, BOTH DAY AND EVENING

BY

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WITH 86 ILLUSTRATIONS

PHILADELPHIA

P. BLAKISTON'S SON & CO.

1012 WALNUT STREET

May 8, 1920

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THE MAPLE PRESS YORK PA

PREFACE

The following pages presuppose a knowledge of the ordinary course of Grammar School Arithmetic.

They are intended:

First.—To review and give drill in the mathematical tools needed by *boys* in the shops during the first year of the Industrial High School.

Second.—To give some of the problems the boys *must* handle in the school shops and *may* have to handle in practical life. Many of the problems have been taken from the shops of the Wm. L. Dickinson High School.

Third.—To introduce the idea of general positive number, its use in formulæ and in simple equations, thereby, incidentally, giving some preparation for the course in algebra. No *formal* approach to algebra, however, is intended.

Fourth.—To give the boy who leaves school through necessity during the first year of his Industrial High School course some of the practical applications of the most used geometrical formulæ.

The manuscript of this book has been used with about thirty different classes under several teachers in the Industrial Department of the Dickinson High School during the past five years. In order that the problems will be expressed in the language and data of the shops and also that they will satisfy the actual mathematical needs of the first year shop work, consultations were held from time to time with the shop teachers.

The drawings were made under the supervision of Mr. Stewart by the pupils of the Industrial Department, most of them by Mr. Rossback of the Junior Class.

The author especially acknowledges his indebtedness to his colleagues Messrs. Burghardt, Steele, Stewart, Loomis and Wagner for valuable suggestions and assistance and also to Mr. Mathewson for his encouragement and help in the preparation of this book.

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INDUSTRIAL ARITHMETIC

LESSON I

The expression 5^2 means 5×5; the expression 3^4 means $3 \times 3 \times 3 \times 3$. In the expression 5^2, 5 is called the base and 2 the exponent. The *exponent* shows how many times the base is to be taken as a factor, that is multiplied by itself.

1. Name the base and the exponent of each of the following expressions, also state the meaning of each:

3^2; 4^3; 5^4; 6^5; 10^2; 10^3; 10^4; 8^4; 7^5.

2. Find the value of each of the above expressions.

3. Expand each of the following:

10^2; 10^3; 10^4; 10^5; 10^6.

4. Note the number of zeros in the expansion of each expression in problem 3. How many zeros are in the expansion of any power of 10?

5. State the power of 10 of each of the following:

1000; 100; 1,000,000; 100,000,000; 10,000; 10; 100,000.

6. Multiply each of the following by 10; by 100; by 1000:

24; 24.3; 2.43; 3.1415.

7. Compare the position of the decimal point in the product with its position in the multiplicand in each example of problem 6. What effect upon the position of the decimal point of a number has multiplying it by 10? By 100? By 10^4? By 10^8? State the principle for multiplying quickly any number by 10 or any power of 10?

8. Write down the value of each of the following:

$.0323 \times 10^3$; 3.141592×10^7; $.000323 \times 10^2$; 32.30×10^4; 3141.592×10^5; $.00323 \times 10^{10}$; 31.415×10^2; $.003141592 \times 10^4$;

$323{,}000 \times 10^5$; 1.732×10^6; $.01414 \times 10^2$; $.507984 \times 10^3$; 52×10^8; $.00005 \times 10^5$; 8976×10^5; $.001 \times 10^4$; 5.00004×10^6; 8.976×10^4.

9. State a quick, easy way for multiplying a number by 20; by 300; by 4000; by 50,000.

10. State the result of multiplying each of the following numbers by 20; by 400; by 5000; by 6000:

123; 3.24; .0122; 83.016; 12,140.

11. Since $25 = 100/4$ then $12 \times 25 = 12 \times 100/4 = 1200/4 = 300$.

That is to multiply a number by 25 we move the decimal point two places to the right and divide the result by 4.

12. Since $33\frac{1}{3} = 100/3$, what is the principle for multiplying a number by $33\frac{1}{3}$?

13. State the principle for multiplying a number by $12\frac{1}{2}$; by $16\frac{2}{3}$; by 50; by $66\frac{2}{3}$; by $11\frac{1}{9}$.

14. Use the above principles and multiply each of the following numbers by 25; by $33\frac{1}{3}$; by $16\frac{2}{3}$; by $12\frac{1}{2}$; by $11\frac{1}{9}$; by 50; and by $66\frac{2}{3}$:

17.28; 1.448; 981; 1892; 1658; 3.1416; 2.0524; 78.6; 4.32; 1214.0678; 3.572.

15. At $12\frac{1}{2}$¢. per ft. what will 1448 ft. of concrete cost? At $16\frac{2}{3}$¢. per ft.? At $11\frac{1}{9}$¢. per ft.?

16. How many feet will a line 12 in. long represent if the scale is 1 in. = $12\frac{1}{2}$ ft.? If 1 in. = $33\frac{1}{3}$ ft.? If 1 in. = 25 ft.?

17. If a cubic foot of iron weighs 480 lb. how many pounds will a plate weigh that contains $16\frac{2}{3}$ cu. ft.? 25 cu. ft.? $12\frac{1}{2}$ cu. ft.? $66\frac{2}{3}$ cu. ft.? 75 cu. ft.? $11\frac{1}{9}$ cu. ft.?

LESSON II

REVIEW

Divide 175.6 by 10; by 100; by 1000.

Compare the position of the decimal point of the quotient with that of the dividend.

State a short easy method for dividing a number by 10 or any power of 10.

1. State the quotient:

56.34 ÷ 10; 76.54 ÷ 1000; 587 ÷ 100; .057 ÷ 100; 563.4 ÷ 100; 893.2 ÷ 10; 68.3 ÷ 1000; .003 ÷ 10; 5.634 ÷ 1000; 8932 ÷ 10,000; .78 ÷ 10,000; .056 ÷ 1000; .789 ÷ 10; .0683 ÷ 100; .06 ÷ 100; .56 ÷ 10.

2. State a short easy method for dividing a number by 20; by 300; by 4000; by 3000; by 80.

3. State the quotient:

248 ÷ 20; 67.5 ÷ 500; 7593 ÷ 5000; 89 ÷ 400; 248 ÷ 40; 7.86 ÷ 200; 759.3 ÷ 5000; 8.9 ÷ 40; 246 ÷ 600; 3675 ÷ 2000; 7.593 ÷ 5000; .89 ÷ 4000; 1728 ÷ 1200; 8958 ÷ 5000; .7593 ÷ 5000; 76 ÷ 1900.

4. At 90¢. per 1000 cu. ft. what will 2486 cu. ft. of gas cost?

5. How many tons in 5240 lb.? In 137,801 lb.? In 756 lb.?

6. At $25 per 1000 what will 12,280 bd. ft. of lumber cost?

7. At $12½ per 1000 find the cost of 16,300 bricks.

8. At $5 per ton, how much must be paid for 12,340 lb. of coal?

9. A gallon of paint will cover 200 sq. ft. of surface two coats. State a rule for finding the number of gallons of paint required to give any surface two coats.

10. Find the number of gallons of paint required for two coats for 2578 sq. ft.; for 3780 sq. ft.; for 160 sq. ft.

11. A quart of grass seed is sufficient for 300 sq. ft. of lawn. How many quarts will be required to seed 2970 sq. ft.?

LESSON III

REVIEW

$$\text{Since } 25 = \frac{100}{4} \text{ then}$$

$$1200 \div 25 = 1200 \div \frac{100}{4} = \frac{1200 \times 4}{100} = 48.$$

That is, *to divide a number by* 25, *move the decimal two places to the left and multiply result by* 4.

1. Divide each of the following by 25:

1660; 72.86; 56; 56.4; 1728; 3564; 3.1416.

2. Study the above and find a short, easy way for dividing a number by $33\frac{1}{3}$; 50; $16\frac{2}{3}$; $11\frac{1}{9}$; $14\frac{2}{7}$; $66\frac{2}{3}$.

3. State the quotient:

$1800 \div 33\frac{1}{3}$; $12.34 \div 12\frac{1}{2}$; $35.81 \div 50$; $1234 \div 50$; $27.16 \div 33\frac{1}{3}$; $35.7 \div 12\frac{1}{2}$; $17.60 \div 14\frac{2}{7}$; $75.16 \div 25$; $87 \div 33\frac{1}{3}$; $3.46 \div 16\frac{2}{3}$; $18.32 \div 11\frac{1}{9}$; $1765 \div 33\frac{1}{3}$; $128 \div 33\frac{1}{3}$; $2468 \div 16\frac{2}{3}$; $175.3 \div 66\frac{2}{3}$; $18.32 \div 14\frac{2}{7}$.

4. In problem 3 change each division sign to a multiplication sign and state product.

5. It is required to cut a 16-ft. bar of steel into pieces $12\frac{7}{16}$ in. long; how many pieces can be cut from the bar, allowing $\frac{1}{16}$-in. waste for each piece in cutting?

6. At $16\frac{2}{3}$¢. per lb. how many pounds of copper wire can be bought for \$18? For \$2400? For \$300?

7. How many taper shanks $24\frac{7}{8}$ in. long can be cut from a round piece 12 ft. long, if $\frac{1}{8}$ in. to each piece is wasted in cutting?

8. A man bought a building lot at $\$16\frac{2}{3}$ per ft. What did he pay for a 68-ft. lot?

9. Find a short easy way for multiplying a number by 99, by 101.

Hint.—$99 = 100 - 1$.

LESSON IV

REVIEW

1. $\frac{3}{4} + \frac{1}{2} + \frac{7}{8} = ?$ $\quad$ $15\frac{15}{64} + \frac{3}{8} + \frac{1}{4} = ?$

$\frac{5}{8} + \frac{1}{2} - \frac{3}{4} = ?$ $\quad$ $\frac{7}{8} + 3\frac{1}{32} - 1\frac{11}{64} = ?$

$1\frac{15}{16} + \frac{1}{8} - \frac{1}{4} = ?$ $\quad$ $1\frac{9}{64} - \frac{1}{8} + \frac{3}{4} - \frac{1}{2} = ?$

$17\frac{17}{32} + \frac{3}{64} + \frac{7}{8} = ?$ $\quad$ $29\frac{29}{32} + \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = ?$

$63\frac{3}{64} + 31\frac{1}{32} + \frac{7}{8} + \frac{3}{4} = ?$

2. $7\frac{7}{16} + 8\frac{3}{8} + 5\frac{15}{16} = ?$

$9\frac{3}{16} + 2\frac{7}{8} - 3\frac{15}{64} = ?$

$10\frac{1}{2} - 8\frac{1}{4} + 2\frac{15}{16} = ?$

$29\frac{7}{8} + \frac{1}{8} - 15 + \frac{1}{2} - 14\frac{1}{2} = ?$

$38\frac{59}{64} + 10\frac{1}{2} - 12\frac{27}{32} + 8\frac{1}{4} = ?$

$15\frac{3}{8} + 8\frac{11}{32} + \frac{15}{64} - \frac{3}{4} = ?$

3. Multiply $15\frac{7}{8}$ by 12; $18\frac{3}{4}$ by 8; $17\frac{3}{8}$ by 8; $28\frac{1}{4}$ by 8.

4. $\frac{7}{8} \times \frac{5}{7} \times \frac{4}{10} = ?$ $\quad$ $\frac{5}{12} \times \frac{3}{4} \times \frac{16}{15} = ?$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = ?$ $\quad$ $\frac{5}{8} \times \frac{16}{25} \times \frac{45}{64} = ?$

$\frac{7}{16} \times \frac{15}{32} \times \frac{32}{45} = ?$ $\quad$ $5\frac{1}{2} \times 3 \times 2 = ?$

$\frac{17}{11} \times \frac{22}{34} \times \frac{3}{5} = ?$ $\quad$ $2\frac{7}{8} \times 5 \times 8 = ?$

$3\frac{7}{12} \times 4\frac{1}{2} \times 3\frac{1}{8} = ?$

5. Divide $1\frac{7}{64}$ by 2; $3\frac{1}{2}$ by $3\frac{1}{7}$; 28 by $3\frac{1}{7}$; $3\frac{1}{3}$ by $2\frac{1}{2}$; $\frac{7}{9}$ by $\frac{3}{5}$.

6. $15\frac{15}{16} \div \frac{3}{8} = ?$ $\quad$ $48\frac{48}{59} \div 22\frac{2}{7} = ?$

$16 \div 5\frac{1}{2} = ?$ $\quad$ $18 \div 16\frac{1}{2} = ?$

$\frac{3}{5} \div 13\frac{13}{64} = ?$ $\quad$ $27\frac{27}{32} \div \frac{1}{4} = ?$

$\frac{7}{16} \div 5 = ?$ $\quad$ $19\frac{19}{32} \div 3\frac{1}{3} = ?$

7. From $15\frac{1}{3} \times 3$ take $5\frac{1}{2} \times 4$ and divide the difference by $6\frac{1}{4}$.

8. Multiply 3.1416 by 26; by 2.6; by .26; by .026.

9. Divide each of the following by 3.1416 and carry the division to two decimal places:

15; 23.48; 178.532; 17; 2; 4.8.

10. Multiply .015625 by $12\frac{1}{2}$; by 25; by $16\frac{2}{3}$; by $33\frac{1}{3}$.

11. Change to decimal fractions:

$\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; $\frac{3}{32}$; $\frac{15}{64}$.

12. Change to common fractions in their lowest terms:

.15; .015625; .0625; .125; .03125; .235.

LESSON V

GENERAL NUMBER

In our previous work all the numbers we have used have had particular values and were represented by a definite symbol. For example the symbol 5 stands for a group of five units. We shall now use other symbols to represent numbers which may have *any values whatever*, or numbers whose values are, as yet, *unknown*, *e.g.*, we speak of a rod a ft. in length meaning *any number of feet*, of x marbles meaning any number of marbles, etc.

1. If John has \$6 and Henry has \$5, together they will have \$6 + \$5.

2. If John has \$a and Henry has \$b, together they will have \$a + \$b.

3. In which of the above problems are numbers used that are represented by particular values? That are represented by any value whatever?

4. If in problem 2, a stands for 10 and b stands for 5, how many dollars have they together? If $a = 7$ and $b = 8$? If $a = 12$ and $b = 10$? If $a = 13$ and $b = 17$?

5. Read each of the following and tell what operation is included:

(*a*) $7 + 5$; 8×3; $9 \div 3$; $8 - 7$; $9 + 5$; $5 + 3$; $9 - 6$; 7×5; $16 \div 4$; $7 - 5$; $8 \div 7$; $8 + 10$.

(*b*) $a + b$; $a - b$; $a \times b$; $a \div b$; $\frac{a}{b}$; $3a$; $3b$; $4c$; $2a + 3c$; $4x \times 3b$; $3 \times n$; $5 \div k$; $\frac{5}{k}$; $7ab$; $9ef$; $(x - 1) \div p$; $a \times c + d$; $n - 5$; $6 + 8b$; $3a - 2b + cd$.

Express with the proper symbol of operation the solution of each of the following:

6. Tom has 10 marbles and Frank has 7 marbles, how many marbles have the boys together? If Tom has a marbles and Frank has r marbles?

7. What is the perimeter of a square if a side is 6 ft.? S ft.? T ft.? X ft.? B ft.? 10 ft.?

8. By how much does 11 exceed 8? 12 exceed 9? 7 exceed 5? 10 exceed k? k exceed 3? e exceed f? r exceed t? y exceed 2p?

9. If the age of a boy is now 16 years, how old was he 7 years ago? 8 years ago? What operation is used to solve this problem?

How old was he a years ago? d years ago? q years ago?

10. At $3 each how many hats can be bought for $18? For $24? For $36? For $a? For $f? For $h?

11. At $a each how many books can be bought for $15? For $13? For $b? For $v? For $x? For $i?

12. At $2 each what will 6 hammers cost? 8 hammers? a hammers? c hammers? k hammers?

13. How many feet longer is a 12-ft. stick than a 10-ft. stick? A 15-ft. stick than a 12-ft. stick? An a-ft. stick than a b-ft. stick?

Make a word problem for each of the following:

14. $7 + 5$; $a + b$; $a - b$; $\$a \times b$; $\$a \div \$G$; $\$a \div \$y$; $3a$; $5b$.

15. What is the sum of 3 ft. and 5 ft.? Of $3a$ and $5a$? Of $7a$ and $6a$? Of $3m$ and $5m$? Of $3a$ and $5a$?

16. What is the difference between 8 books and 5 books? Between $8x$ and $3x$? Between $10l$ and $3l$? Between $8s$ and $5s$? Between $7r$ and $5r$?

17. $5a + 6a - 2a + 4a = (\ )a$; $12y - 8y + 10y - 5y = ?$; $3b + 7b - 5b + 8b = (\ )b$; $3k + 4k - 5k - 2k = ?$; $4x + 5x - 8x + 10x = ?$; $8s + 7s - 10s - 5s = ?$; $8\frac{1}{4}l + 2\frac{1}{4}l -$

$7\frac{1}{2}l - 2\frac{1}{3}l = ?$ $3m + 10m - 8m - 2m = ?$; $3\frac{1}{2}t + 7t + 8\frac{1}{2}t - 4r = ?$; $9\frac{1}{3}d - 7\frac{1}{3}d + 4\frac{2}{3}d - 2d = ?$

18. Find the value of each of the following expressions if $a = 1, b = 2, c = 3$:

$a + b$; $c - d$; $c + b - a$; $(2c + b) \div 2$; $(8a - 2) \div b$; $c + b + a$; $a \times b \times c$; $b \times c - a$; $2c \div b + 5$; $4c + 5 - 8\frac{1}{2}b$; $3\frac{1}{3}c - 5b$; $4\frac{1}{4}b - 2a$; $\frac{c}{2} + \frac{1}{b}$; $\frac{1}{c} + \frac{1}{b}$; $\frac{1}{c} - \frac{1}{b}$.

19. The expressions ab, a·b, a × b all mean that a is to be multiplied by b.

The expression a(b + c + d) means that b, c and d are to be added and the sum multiplied by a. When 2 or more numbers have no sign of operation between them like abcd, it is to be understood that the numbers are to be multiplied.

LESSON VI

FORMULÆ

The statement of a rule by means of general numbers and other mathematical symbols is called a *formula*. For example, the rule for finding the area of a circle is, multiply the square of the radius by 3.1416. This rule stated by means of a formula, if S = area of circle, π = 3.1416 and r radius of the circle, is $S = \pi r^2$.

1. If $\frac{a}{b}$ and $\frac{c}{d}$ be any two fractions, state by means of a formula the rule for the multiplication of two fractions; for the division.

2. State as a formula the following: The area of a rectangle is equal to its base multiplied by its altitude. Let b = base and h = altitude.

3. If b = the area of the base, h = the altitude and v = the volume of a cone, from $v = \frac{1}{3}bh$, state in words the rule for finding the volume of a cone.

4. The formula for finding volume (v) of a sphere whose radius is r is $v = \frac{4}{3}\pi r^3$. Find volume of a sphere whose radius is 5, 6, 8, 10.

5. L = length of open belt in feet (approximate). D = distance between centers of pulleys in feet. R and r = radii of two pulleys in feet.

$$L = 3\tfrac{1}{4}(R + r) + 2d.$$

The distance between the centers of two pulleys is 20 ft. and the radii of the pulleys 18 in. and 11 in. Find the approximate length of the open belt required for the pulleys.

6. The approximate distance a body will fall from rest in any number of seconds is given by $D = 16t^2$.

Find the distance a body will fall from rest in 4 sec.; 10 sec.; 15 sec.; 1 min.

7. C = Circumference of a circle and r = its radius.

$$C = 2\pi r.$$

What is the velocity of a point on the rim of a wheel, radius 3 ft. making $12\frac{1}{2}$ revolutions per second.

LESSON VII

ANGLES AND POLYGONS

1. The figure BAC is called the angle BAC. The point A is called the vertex of the angle and the straights AB and AC are called the sides of the angle. In reading an angle the vertex is always read between the other two letters, as angle BAC; writ-

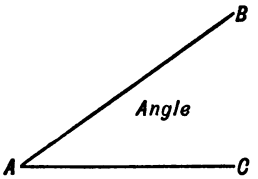


FIG. 1.

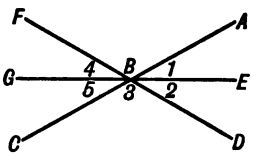


FIG. 2.

ten ∠BAC. An angle is often named from its vertex letter only, as the above is called the angle A; written ∠A.

2. Read the angles of Fig. 2.

These angles may be called ∠1, ∠2, ∠3, etc.

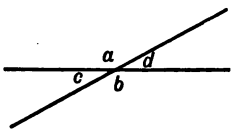


FIG. 3.

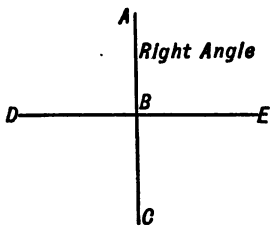


FIG. 4.

3. The angles a and b are called vertical angles. Name another pair of vertical angles in the above figure. Name pairs of vertical angles in the figure for example 2.

4. The angles a and d are called adjacent angles. Name other pairs of adjacent angles in the above figures.

5. *Two angles are equal if their sides can be made to coincide.*

6. If two straights so intersect that any pair of adjacent angles formed are equal, the straights are said to be perpendicular to each other and the angles formed are right angles.

If the $\angle ABE = \angle DBA$ (Fig. 4), then the straight AC is perpendicular to the straight DE and the angles formed are right angles. Name the right angles in Fig. 4.

7. Right angles and perpendiculars are often constructed by means of the T square or a right triangle.

8. An angle is often measured by the number of degrees it contains. An angle of one degree (1°) is one of the 90 equal angles into which the right angle can be divided.

How many degrees in one right angle? Two? Three? Four? ½ right angle? ⅓? 1½? ¾? ⅙?

A protractor is used to measure an angle in degrees.

9. If the sides of the angle lie in the same straight the angle is called a straight angle, *e.g.*, $\angle DBE$ is a straight angle (Fig. 4).

Name other straight angles.

How many degrees in a straight angle? How many right angles?

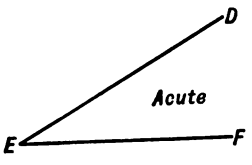


FIG. 5.

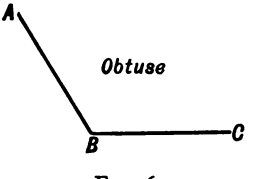


FIG. 6.

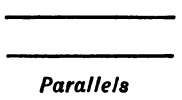
10. An angle less than a right angle is called an acute angle.

11. An angle greater than a right angle but less than a straight angle is called an obtuse angle.

12. Straights in the same plane that have no common point are called parallel lines.

Parallel lines are constructed by means of the parallel ruler or the T square.

13. A curved line is a line no part of which is straight.



Parallels

FIG. 7.

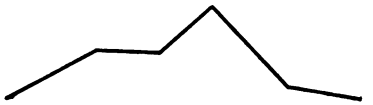
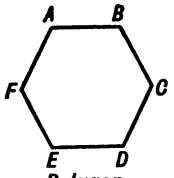


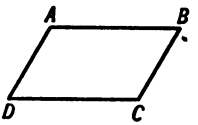
FIG. 8.

14. A line not straight but no part of which is curved is called a broken line (Fig. 8).



Polygon

FIG. 9.



Parallelogram

FIG. 10.

15. If the end points of a broken line coincide the figure formed is called a polygon.

AB, BC, CD, etc., are the sides of the polygon. The angles FAB, ABC, etc., are the angles of the polygon.

The broken line is called the *perimeter* of the polygon.

Exercise.—If each side of the above polygon is 7 in. what is the length of its perimeter? If 5 in.? If 10 in.? If 3 ft.? If 5 ft. 4 in.?

16. A polygon of three sides is a *triangle.* A polygon of four sides is a *quadrilateral.*

17. If the opposite sides of a quadrilateral are parallel the figure is called a parallelogram.

Exercise.—How many sides has a parallelogram? Why? Can the sides of a parallelogram be curved? Why?

18. Facts relating to a parallelogram: Draw a parallelo-

gram and letter it A, B, C, D. Use the compasses and compare AB with CD. Are they equal?

Compare AD with BC. Are they equal?

Draw another and make the same comparison.

Compare the opposite angles in each parallelogram.

The above exercises illustrate the following principles:

1. The opposite sides of a parallelogram are equal.

2. The opposite angles of a parallelogram are equal.

Exercise.—If in the parallelogram ABCD, AB = 10 in., CD = ? Why? AD = 3 in., BC = ? Why? CD = 15 ft., AB = ? Why? BC = 3 ft. 4 in., AD = ? Why? B = 78°, D = ? C = 102°, A = ?

The Rectangle

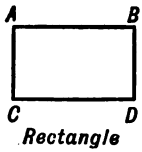


Fig. 11.

19. A parallelogram having one of its angles a right angle is called a rectangle.

Exercise.—Are the opposite sides of a rectangle equal? Why? Are the opposite angles of a rectangle equal? Why?

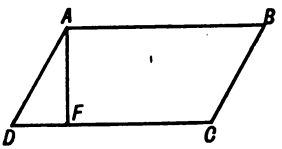


Fig. 12.

At least how many of the angles of a rectangle are right angles?

Find by means of a right angle whether the other angles of a rectangle are right angles.

20. Any one of the sides of a parallelogram is called its base; *e.g.*, AB, or BC or AD or CD is a base of the parallelogram.

Name the bases of the rectangle in example 19.

21. The perpendicular from one side of a parallelogram to the opposite side is the altitude of the parallelogram, *e.g.*, AF in the figure for example 20.

Name the altitude of the rectangle in example 19 if DC is the base; if AD is the base.

THE SQUARE

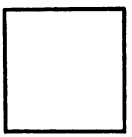


FIG. 13.

22. A rectangle with a pair of intersecting sides equal is a square.

Exercise.—Is a square a parallelogram? Why?

How many degrees in at least one angle of a square? Why? In each angle of a square? Why? Are the sides of a square equal? Why? Name the bases and altitudes of the above square. If one base of a square is 2 in., what is the length of its altitude? Draw a square one of whose sides is 12 in., letting 1 in. = 6 in. Draw a rectangle with base 20 ft. and altitude 4 ft. (scale 1″ = 4′ 0″.) Draw a 1-in. square, or 1 sq. in.; a 1-ft. square or 1 sq. ft.

LESSON VIII

MEASUREMENTS

Measuring a line consists in finding the number of standard units of length it contains.

There are many standard units of length in common use, such as the inch, foot, yard, etc.

The number of standard units of length a line contains is the *length* of the line *in terms of that unit; e.g.*, a line is 5 ft. long if it contains the foot five times.

In the wood working shops the ruler used for measuring lengths is divided into inches and each inch divided into 16 equal parts, the wood worker thus measures to $\frac{1}{16}$ in. In the machine shops one of the instruments used enables the machinist to measure to $\frac{1}{1000}$ in. This instrument is called a micrometer and is used for measuring fractional parts of an inch. This instrument records its measurements in the decimal scale instead of in terms of common fractions. The ruler of the wood worker reads $\frac{1}{8}$ in. whereas the micrometer of the machinist would read .125 in. for the same length. The ordinary micrometer registers accurately to .001 in. The experienced machinist can measure very closely with it to .0001 in.

Wood Working Measurements

1. Measure as accurately as possible the top of your desk.

2. Find the number of feet in the length of the schoolroom; the number of yards.

3. Measure the length and width of your schoolroom. How many feet of baseboard are required for the room, making deductions for all openings in the baseboard?

4. Find the cost of the chalk trays of the blackboards in your schoolroom at $12\frac{1}{2}$¢. per ft.

5. Determine the number of feet of moulding in the panels of the doors of the schoolroom and its cost at $5\frac{1}{2}$¢. per ft.

6. How many strips of floor moulding 16 ft. long will be necessary for the schoolroom?

7. How many feet of picture moulding will be required for the room if the moulding is at the top of the walls? If 3 ft. from the top of the walls?

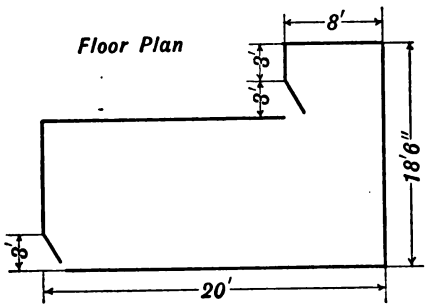


FIG. 14.

8.

The above is a floor plan.

(*a*) Find cost of the floor moulding for it at $8\frac{1}{2}$¢. per ft.

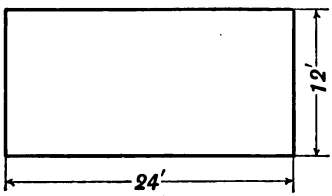
(*b*) Find cost of the baseboard at 20¢. per ft.

9. What is the perimeter of a rectangular room M ft. long and R ft. wide? S ft. long and 12 ft. wide? K ft. wide and 2K ft. long?

LESSON IX

DRAWING MEASUREMENTS

The inch of the scale used for measuring in the drawing room is divided into 32 equal parts. To what fraction of an inch can the draughtsman measure? In the drawing room working plans for the machinist, the carpenter, the pattern maker, etc., are made. These drawings are seldom made full size but are drawn to a scale, that is, each inch of the drawing represents 1 ft. or 2 ft., etc., of the actual size of the object drawn. When making drawings it frequently happens that a line must be drawn whose length contains $\frac{1}{32}$ in. or even $\frac{1}{64}$ in. In ordinary drawings $\frac{1}{64}$ in. is neglected, no attempt being made to draw any line shorter than $\frac{1}{32}$ in.



Scale $\frac{1}{4}'' = 4'\ 0''$

FIG. 15.

Notice that the plan has marked on it the actual lengths of the lines represented, but the lengths of the drawing are made in accordance with the scale selected.

Exercises

1. If the scale is $1'' = 3'\ 0''$, how long a line must be drawn to represent each of the following:

9 ft. 0 in.; 12 ft. 0 in.; 15 ft. 0 in.; 6 ft. 0 in.; 18 ft. 0 in.; 100 ft. 0 in.; 8 ft. 6 in.; 7 ft. 5 in.; 15 ft. 8 in.; 18 ft. $\frac{5}{12}$ in.; 25 ft. $\frac{7}{12}$ in.; 9 in.; 15 in.; 2 in.?

2. If the scale is 1″ = 0′. 8″, determine the length of each of the following for the drawing:

1 in.; $\frac{1}{2}$ in.; $\frac{7}{8}$ in.; $2\frac{1}{4}$ in.; $3\frac{1}{8}$ in.; 5.5 in.; 6.25 in.; 4.125 in.

3. In the above figure what should be the length of each line?

4. Fill in the following, the scale being 1″ = 3″.

Length of line in drawing	Actual length represented by drawing
6 in.	?
9 in.	?
2 in.	?
$3\frac{1}{8}$ in.	?
$5\frac{5}{32}$ in.	?
?	16.5 in.

5. Calculate the length of each line on the drawing for the miter box if the scale is half size; if one-quarter size. (Fig. 16.)

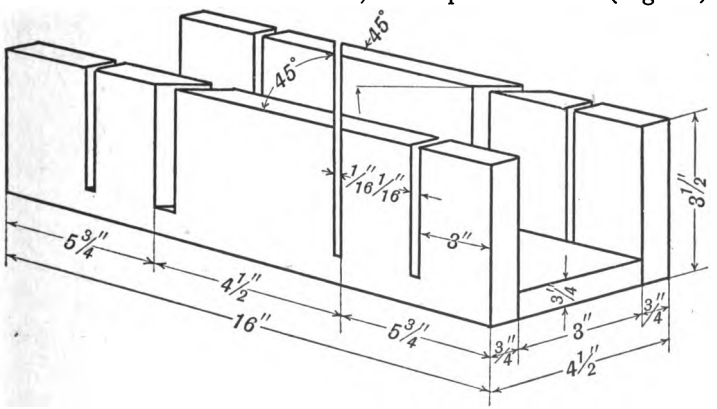


FIG. 16.

6. What is the length of each line in the drawing for the mortar box, if the scale is $\frac{1}{8}$″ = 0′. 1″? (Fig. 17.)

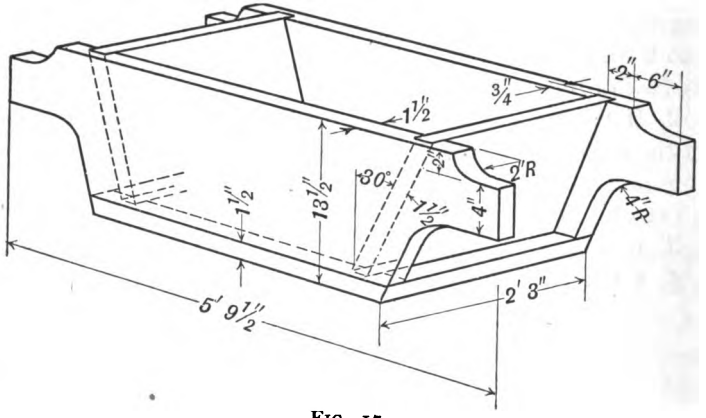


FIG. 17.

7. The scale for the saw horse is 3″ = 1′ 0″. Calculate the length of each line for the drawing. (Fig. 18.)

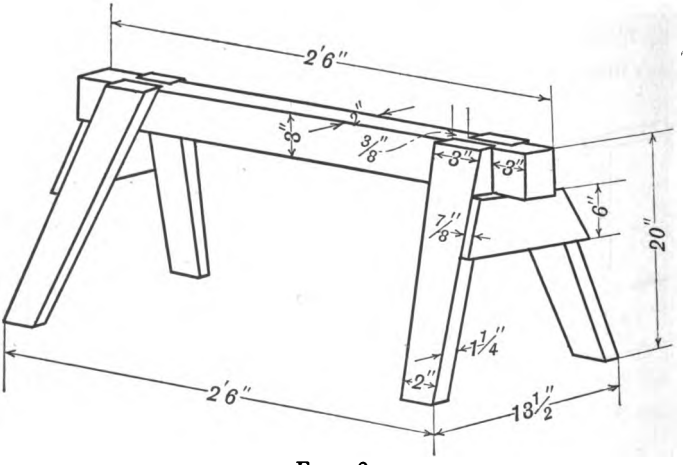


FIG. 18.

8. The side of a filing cabinet is to be 14 in. long 10½ in. high, the stile and rail each 1½ in. wide. Calculate the dimensions for a drawing with scale ⅛″ = 1′ 0″; ¼″ = 1′ 0″.

LESSON X

SCREW THREADS

1. If a cylinder is spirally grooved or has a thread wound spirally around it, a screw is formed. The nature of a screw depends upon the diameter of its body, the number of threads per inch, and the shape of the thread.

2. Upon the number of threads per inch depend the pitch of the screw and the lead of the screw. *The pitch* of a screw is the distance between the top of one thread and the top of the next. For example, if a screw has 8 threads per in., the pitch is evidently $\frac{1}{8}$ in.

3. *The lead* of a screw is the distance it advances in one revolution. In a single-thread screw with 10 threads per in. the lead is $\frac{1}{10}$ in.

Exercises

(The following problems refer to screws of single thread.)

1. A screw has 12 threads per in. What is its pitch? Its lead?

2. Find the pitch and lead for the following number of threads per inch:

3; 4; 5; 6; 8; 11; 20; 13; 40

3. A screw that has 40 threads per inch will advance how far in one complete revolution of the screw?

4. Find the number of threads per inch for the following leads:

$\frac{1}{4}$ in.; $\frac{3}{8}$ in.; $\frac{1}{2}$ in.; $\frac{1}{40}$ in.; $\frac{7}{8}$ in.; $\frac{3}{4}$ in.; $\frac{1}{20}$ in.

5. What is the pitch of a screw that has a threads per in.? Its lead?

6. Does the lead of a single-thread screw equal its pitch? Why?

7. If the lead of a screw is $\frac{1}{4}$ in., what distance will it advance in $\frac{1}{2}$ revolution? $\frac{1}{4}$ revolution? $\frac{1}{25}$ revolution? $\frac{3}{5}$ revolution?

8. A screw has 40 threads per in.; what distance will it advance in $\frac{1}{25}$ revolution? $\frac{2}{25}$? $\frac{1}{5}$? $\frac{15}{25}$? $\frac{24}{25}$? $\frac{17}{25}$? 2? 10? 15? 20? 40?

9. Express as a decimal, correct to three places, the lead of each of the following number of threads per inch:

40; 20; 10; 8; 12; 7

10. Express each of the results in problem 8 as a decimal correct to three places.

LESSON XI

MACHINE SHOP MEASUREMENTS

The measuring instruments of the machine shop are a 6-in. scale graduated on one edge to $\frac{1}{64}$ in. and the micrometer. The 6-in. scale is used for rough measurements and the micrometer for more accurate measurements. The readings of the micrometer, as already stated, are in the decimal scale.

The micrometer is essentially a screw with 40 threads per in. What is its lead? How far in the decimal scale will it advance for $\frac{1}{25}$ of a revolution? How then is .001 in. measured with the micrometer? .002 in.? .006 in.? .015 in.? .026 in.?

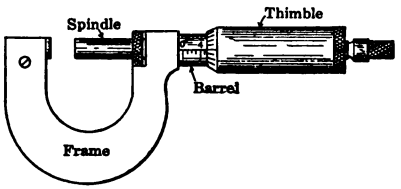


FIG. 19.—A micrometer screw.

The names of the different parts of a micrometer are given in the figure.

The spindle is made to move backward and forward within the barrel by turning the thimble, to which the spindle is fastened. The concealed end of the spindle is a screw containing 40 threads to the inch. On the edge of the thimble are 25 divisions equally distant. By turning one division of the thimble, the spindle is made to move through $\frac{1}{25}$ of $\frac{1}{40}$ in. or

.001 in. Each revolution of the thimble is indicated on the barrel by means of a small mark and every fourth revolution of the thimble by a longer mark. The barrel is thus divided for the space of 1 in.

Exercises

1. How many divisions of the thimble must be turned in order to have the micrometer measure each of the following:

.003 in.; .005 in.; .010 in.; .025 in.; .018 in.; .075 in.; .125 in.?

2. What part of an inch is the distance between any pair of consecutive small divisions of the barrel? Between any pair of consecutive large divisions of the barrel?

3. What part of an inch is the distance between 6 consecutive divisions of the barrel? 8? 10? 15? 20? 40? 41?

4. How many revolutions of the thimble for each of the following:

.025 in.; .050 in.; .075 in.; .150 in.; .200 in.; .3 in.; .5 in.?

5. What decimal part of an inch do the following barrel readings represent:

5 large divisions; 3 large divisions; 6 large divisions; 10 large divisions; 4 large divisions; 4 large divisions and 2 small divisions; 3 large divisions and 3 small divisions; 2 small divisions?

6. Express as the decimal part of an inch the following readings:

5 large and 3 small of the barrel and 12 of the thimble; 3 large and 2 small barrel, and 16 thimble; 1 large of the barrel and 8 thimble; 6 large and 3 small barrel, and 14.5 thimble; 9.5 thimble.

LESSON XII

DECIMAL EQUIVALENTS

As we have already learned the machine shop's fractional measurements are in multiples of $\frac{1}{64}$ in.; also, if these measurements are to be accurate the micrometer is used. Hence, in order to measure $\frac{1}{64}$ in. its equivalent as a decimal must be known. Why?

Show that $\frac{1}{64} = .015625$. (Learn this result.)

In setting the micrometer for ordinary work three decimal places only are considered, that is $\frac{1}{64} = .015$. If it is required to set the micrometer for $\frac{1}{32}$ or $\frac{2}{64}$ we consider $\frac{1}{64} = .0156$, because to get $\frac{2}{64}$ we multiply $\frac{1}{64}$ by 2, $\frac{1}{32} = \frac{2}{64} = .031$; that is $\frac{1}{64} = .0156$ whenever it is to be multiplied by any number.

By means of the following it is possible to state quickly the decimal equivalent of any number of 64ths.

$$\frac{1}{2} = .5$$
$$\frac{1}{4} = .25$$
$$\frac{1}{8} = .125$$
$$\frac{1}{64} = .015625$$

Express as a decimal $\frac{17}{64}$.

Solution.—$\frac{17}{64} = \frac{16}{64} + \frac{1}{64} = \frac{1}{4} + \frac{1}{64} = .25 + .015 = .265$.

Express as a decimal $\frac{63}{64}$.

Solution.—$\frac{63}{64} = \frac{64}{64} - \frac{1}{64} = 1 - .0156 = .984$.

Express as a decimal $\frac{11}{64}$.

Solution.—$\frac{11}{64} = \frac{8}{64} + \frac{3}{64} = \frac{1}{8} + \frac{3}{64} = .125 + .046 = .171$.

1. Express the decimal equivalent to three places of each of the following:

$\frac{1}{64}$; $\frac{1}{32}$; $\frac{3}{64}$; $\frac{1}{16}$; $\frac{5}{64}$; $\frac{11}{64}$; $\frac{7}{8}$; $\frac{5}{16}$; $\frac{5}{32}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{17}{64}$; $\frac{9}{32}$; $\frac{15}{32}$; $\frac{29}{64}$; $\frac{31}{64}$; $\frac{33}{64}$; $\frac{55}{64}$; $\frac{57}{64}$; $\frac{9}{16}$.

2. What error is made in considering $\frac{3}{16} = .186$? $\frac{59}{64} = .921$? $\frac{17}{32} = .531$?

3. Obtain a piece of cardboard or stiff paper. Rule it as shown below making the left-hand section long enough to contain the 64ths from 1 to 32 and the right-hand section the 64ths from 33 to 64. Fill in the blanks as illustrated. Carry the decimals to six places. Keep it for future use.

64 ths	32 ds	16 ths	8 ths	4 ths	Dec.	64 ths	32 ds	16 ths	8 ths	4 ths	Dec.
1					.015625	33					.515625
2	1				.031250						
4	2	1			.062500						
16	8	4	2	1	.250000						

LESSON XIII

REVIEW

1. In the formula W = F.S., F = 240 and S = 25. Find W.

2. If H.P. = $33\frac{1}{3}$ and V = $16\frac{2}{3}$ find the value of E in $E = \frac{33{,}000 \text{ H.P.}}{V}$.

3. Given that $T = \frac{WV^2}{32.2}$, find the value of T when W = $16\frac{2}{3}$ and V = 26.

4. The barrel reading of the micrometer when used to measure the diameter of a piece of round stock is 4 large divisions and 1 small division. What is the diameter of the stock expressed as a common fraction?

5. What is the pitch and also the lead of each of the following screws:

25 threads per in.; 14; 18; 22; 38?

6. With a scale of $\frac{3}{8}$ in. = 0 ft. 1 in., what is the length of the drawing to represent a line 3 ft. $5\frac{1}{4}$ in.; 8 ft. $6\frac{3}{8}$ in.; 7.5 in.; $\frac{1}{8}$ in.?

7. The fractional dimensions on the drawings for a machinist are usually expressed as decimals. Express the following dimensions suitable for a machine drawing:

$2\frac{15}{16}$ in.; $3\frac{15}{64}$ in.; $5\frac{7}{8}$ in.; $\frac{57}{64}$ in.

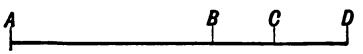


FIG. 20.

8. If in the above figure AB = 12 ft. 6 in., BC = 3 ft. 9 in., CD = 4 ft. 6 in., what is the length of AD?

9. If AD = 10 ft.; AB = 2 ft. 3 in. and CD = 5 ft. 9 in.; BC = ?

10. If BC = 2AB and CD = 3AB and AD = 18 ft., find AB, BC and CD.

11. If BC = 3.125 in., CD = 4.265 in. and AD = 12 in.; AB = ?

LESSON XIV

AREA OF A RECTANGLE

Measuring a surface consists in finding the number of squares the surface contains.

There are many standard units of surface in common use, such as the square inch, square foot, square yard, etc.

Problems

1. Find the number of square feet in a rectangle of base 10 ft., altitude 4 ft.

How many square feet in the row along the base? How many such rows? How many square feet in the rectangle?

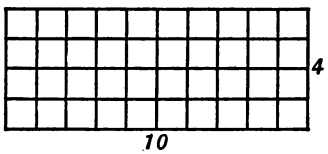


FIG. 21.

2. Find the number of square feet in each of the following rectangles; also explain as in problem 1:

Altitude 5 ft., base 12 ft.; altitude 18 ft., base 10 ft.; altitude $4\frac{1}{2}$ ft., base 18 ft.; altitude $3\frac{1}{3}$ ft., base 9 ft.; altitude 4 ft., base 8 ft. 6 in.; altitude 4 ft., base 8 ft. 6 in.; base 16 ft., altitude 5 ft. 3 in.; altitude 5 ft. 6 in., base 24 ft.; base 10 ft. 10 in., altitude 12 ft. 6 in.; base b ft., altitude c ft.; base x ft., altitude y ft.; base m ft., altitude n ft.; base k ft., altitude r ft.

3. State the principle you have used in solving each of the above. Express the principle as a formula.

The number of squares in a surface is called the area of the surface in terms of the square. The area of problem 1 is 40 sq. ft.

Exercises

1. Find the number of square feet in the floor of a room 15 ft. × 14 ft.

2. How many square feet in the surface of a floor 14 ft. 7 in. long and 12 ft. 5 in. wide?

3. Find the number of square feet in the ceiling, floor and walls of a room 24 ft. × 15 ft. × 10 ft.

4. Find the number of square feet in the surface of a corridor 200 ft. long, 15 ft. high and 12 ft. wide.

5. Find the number of square feet in the doors of your schoolroom; in the windows.

6. Measure the chalk box and find the number of square inches its surface contains.

7. If the height of your schoolroom is 15 ft., find the number of square feet in its surface.

8. Find the number of square feet of plastering in the schoolroom.

9. A box a ft. long, b ft. wide and c ft. high contains how many square feet in its surface?

10. Find the area of the floor plan in lesson 9.

11. Assuming 200 sq. ft. of surface can be covered with two coats with 1 gal. of paint, how many gallons will be required to paint the walls of your schoolroom?

12. What decimal part of an acre in a building lot 86 ft. × 155 ft. 1A = 43,560 sq. ft.

LESSON XV

REVIEW

1. Find the area of the square one of whose sides is:

5 ft.; 6 ft.; 7 ft.; 10 ft.; 12 ft.; 15 ft.; 20 ft.; 25 ft.; 24 ft. 6 in.; 23 ft. 2 in.; 28 ft. 6 in.; 15 ft. 4 in.

2. If a is the side of a square and S is its area what formula will represent its area?

3. Draw on the blackboard a square foot, a square yard. Then fill in the following table and learn it:

—— sq. in. in 1 sq. ft.

—— sq. ft. in 1 sq. yd.

4. How many square inches in 8 sq. ft.? 9 sq. ft.? 15 sq. ft., 98 sq. in.? 18 sq. ft., 75 sq. in.? 3 sq. yd., 4 sq. ft., 80 sq. in.? 50 sq. yd., 2 sq. ft., 85 sq. in.?

5. If you are given the product of two numbers and one of the numbers, how will you find the other number?

Is the area of a rectangle the product of two numbers?

When you know the area of a rectangle and the length of its base, how will you find its altitude?

Let S = area of a rectangle, h its altitude and b its base; then state as a formula your answer to the last question. Also state the formula for its base in terms of S and H.

6. Find the base or altitude of each of the following rectangles:

S = 500 sq. in.; b = 10 in.; S = 750 sq. yd.; h = 15 yd.

S = 1200 sq. ft.; h = 20 ft.; S = 750 sq. yd.; h = 15 ft.

S = 1728 sq. ft.; b = 12 ft.; S = 750 sq. yd.; b = 24 in.

7. How wide must a floor 35 ft. long be to contain 700 sq. ft.?

8. I wish to cut from a board 8 in. wide a piece containing 370 sq. in. How long a piece must I cut?

9. If 20 workmen are placed at a bench 3 ft. wide, all on the same side of it, how long must the bench be if each workman is allowed 12 sq. ft.?

10. Find the length of the perimeter of a rectangle if its area is ab sq. ft., and its altitude is a ft.?

LESSON XVI

SQUARE ROOT

The square root of a given number is *the number*, which when squared or multiplied by itself, will produce the given number; *e.g.*, 4 is the square root of 16, because $4^2 = 16$.

Exercise.—What is the square root of each of the following:

25; 100; 144; 36; 64; 16; 81; 169; $^{25}/_{36}$; $^{49}/_{81}$; $^{100}/_{144}$; $^{169}/_{225}$; $^{4}/_{9}$; A^2; B^2; N^2; P^2?

Numbers whose square roots can be expressed by means of the digits only are called perfect squares.

Exercise.—Name all the perfect squares between 0 and 300.

The square root of a number not a perfect square is approximately expressed by means of decimal fractions. Thus $\sqrt{5} = 2.236$ approximately.

Between what two whole numbers does the square root of each of the following numbers lie:

26; 38; 45; 136; 175; 52; 70; 84?

Exercises

1. Find the square root of:

1369; 3136; 65,536; 98,596; 277,729; 4096; 11,664; 6561; 9216; 9,339,136; 7,387,524; 1,827,904; .0841; .061009; 4907-.0025.

2. Find the side of a square whose area is 225 sq. ft.

3. I desire to make a square platform whose area is 600 sq. ft. Find the length of the side of the platform so that the error shall be less than $\frac{1}{8}$ in.

4. It is required to construct a square opening in the walls of a room for a ventilator. The opening must contain 600

sq. in. Find the side of the opening so that the error shall be less than $\frac{1}{8}$ in.

5. A glazer is required to cut a pane of glass in the form of a square that shall contain 625 sq. in. What size must he cut the glass?

LESSON XVII

REVIEW

1. Which determines the shape of a figure, 2 sq. ft. or 2 ft. square? 5 sq. in., or 5 in. square?

2. What is the difference in square feet between 10 square feet and a 10-ft. square?

What is the difference in square inches between a 6-in. square and 6 sq. in.?

3. How long a board will it take to fill up a rectangular hole of 160 sq. in. and 10 in. wide?

4. How many boards 10 in. wide and 12 ft. long will be required to build a solid board fence 5 ft. high and 48 ft. long if the boards are nailed lengthwise?

5. It is required to build a walk 96 ft. long and 2 ft. 8 in. wide of boards 16 ft. long and 10 in. wide. How many boards will be required if laid lengthwise and how wide a space must be left between the boards?

6. In a room 12 ft. × 15 ft. is a rug 9 ft. × 12 ft. How many square feet of floor between the rug and the walls of the room?

7. How many square inches in a picture frame 15 in. × 9 in. if the frame is 2 in. wide? $2\frac{1}{2}$ in. wide? $2\frac{1}{4}$ in. wide?

8. $15\frac{1}{2}$ sq. ft. = sq. in.? $18\frac{3}{4}$ sq. ft. = sq. in.? $9\frac{2}{3}$ sq. yd. = sq. ft.? $17\frac{1}{4}$ ft. = in.?

9. Extract the square root of 2, of 3, carry the result to three decimal places.

10. $\frac{15}{16} \div \frac{3}{8} = ?$ $17\frac{1}{5} \div 3\frac{1}{3} = ?$ $2\frac{2}{7} \div 3\frac{1}{2} = ?$ $\frac{5}{7} \div \frac{4}{9} = ?$ $3\frac{1}{3} \div 2\frac{1}{5} = ?$ $7\frac{1}{8} \div 3\frac{1}{3} = ?$

LESSON XVIII

FRACTIONAL REVIEW

1. $3\frac{1}{2} \times 4\frac{1}{4} \times \frac{1}{6} = ?$ $\quad 5\frac{1}{6} \times 3\frac{1}{3} \times 4\frac{1}{4} = ?$

$4\frac{1}{5} \times 3\frac{1}{8} \times \frac{3}{4} = ?$ $\quad 5\frac{1}{5} \times 6\frac{2}{3} \times 5\frac{1}{4} = ?$

2. $\frac{5}{12} \times 14\frac{3}{4} \times \frac{2}{3} = ?$ $\quad 17\frac{1}{2} \times 5\frac{1}{8} \times 10\frac{1}{3} = ?$

$144 \times 5\frac{1}{4} \times 7\frac{1}{3} = ?$ $\quad 1728 \times 4\frac{1}{6} \times 7\frac{1}{2} = ?$

3. $(\frac{5}{4} + \frac{9}{8} + \frac{16}{15}) - (\frac{1}{4} + \frac{1}{8} + \frac{1}{15}) = ?$

4. $(\frac{7}{8})^2 \times (\frac{3}{4})^3 \times (\frac{2}{3})^4 = ?$

5. $\dfrac{\frac{3}{11} + \frac{3}{16}}{\frac{3}{22} + \frac{3}{8}} = ?$

6. $\dfrac{(\frac{3}{4} + \frac{5}{6} + \frac{7}{8}) \times (\frac{4}{5} + \frac{6}{9} + \frac{9}{10})}{3\frac{2}{5}} = ?$

7. $\frac{1}{4}$ of $\dfrac{1\frac{1}{7}}{2} \times \dfrac{2\frac{1}{7}}{3} \times \dfrac{3\frac{1}{7}}{4} \times \dfrac{4\frac{1}{7}}{5} = ?$

8. $\frac{9}{10} \times \frac{18}{35} \times \frac{7}{24} \times \frac{16}{27} = ?$

9. $(4\frac{7}{15} + 8\frac{5}{21} + 4\frac{7}{11}) \div (5\frac{2}{7} + 5\frac{4}{5} - \frac{2}{3}) = ?$

10. $\left(\dfrac{\frac{1}{7} + \frac{2}{11}}{8\frac{1}{3}} + \dfrac{7\frac{7}{9}}{6\frac{5}{12}}\right) \div 4 \times \dfrac{5}{7\frac{7}{9}} = ?$

11. $\dfrac{(1\frac{1}{4} \times 1\frac{2}{7} + \frac{1}{3}) \text{ of } (2\frac{1}{4} + 1\frac{3}{28} \times 2)}{(1\frac{3}{28} \text{ of } 2\frac{1}{3} \text{ of } 2\frac{1}{4}) - (1\frac{1}{4} \text{ of } 1\frac{2}{7})} = ?$

12. What number multiplied by $\frac{7}{11}$ of $\frac{9}{14}$ of $29\frac{1}{3}$ will give $102\frac{2}{5}$ for the product?

13. If the rent of $5\frac{1}{12}$ acres of land is $\$21\frac{2}{3}$, what will be the rent of $19\frac{9}{16}$ acres of the land at the same rate?

14. How many taper shanks each $2\frac{1}{4}$ in. long can be cut from $33\frac{1}{4}$ ft. of stock allowing $\frac{1}{8}$ in. to each piece for waste in cutting?

LESSON XIX

AREA OF PARALLELOGRAMS

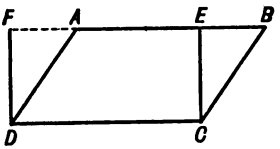


FIG. 22.

1. Draw a parallelogram ABCD on a piece of cardboard or stiff paper. Draw CE perpendicular to AB. Cut off the triangle CEB and place in the position of FAD. What is the name of the parallelogram CEFD? Has it the same number of squares as the parallelogram ABCD? Has it the same base and altitude? How do you find the number of squares in CEFD? How then in ABCD?

2. Find the number of square units in each of the following parallelograms:

Base 10 ft., altitude 8 ft.; altitude 17 ft., base 20 ft.; base 10 ft., altitude 8½ in.; altitude 64 ft., base 25 ft.; base 16½ in., altitude 8½ in.; altitude 75 ft., base 33⅓ ft.; base 10 ft., altitude 5 ft. 9 in.; altitude 13 ft. 8 in., base 12 ft. 6 in.; base b ft., altitude a ft.; altitude h ft., base m ft.

3. The expression ab sq. ft. stands for the number of square feet of what two figures?

4. Find the missing parts of the following parallelograms:

40 sq. ft., altitude 10 ft.; 75 sq. ft., base 8 ft.; 125 sq. ft., altitude 15 ft.; altitude 18 ft., 180 sq. ft.

LESSON XX

AREA OF TRIANGLES

1. A triangle is a polygon of three sides. Any one of the sides is a base of the triangle. The vertex of any angle is a vertex of the triangle. How many bases has a triangle? How many vertices?

The perpendicular from any vertex to the opposite side is an altitude of the triangle. How many altitudes has a triangle? Draw a triangle and its altitudes.

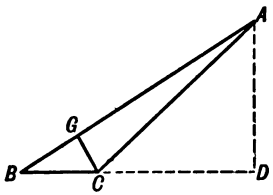


FIG. 23.

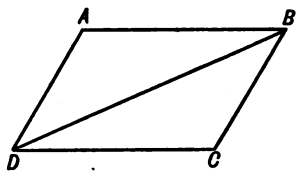


FIG. 24.

2. The line BD is called a diagonal of the parallelogram. Name another diagonal of the figure.

3. Draw on and cut out from a piece of cardboard or stiff paper a parallelogram ABCD. Then cut along the diagonal BD. Are the two triangles formed equal? A triangle with the same base and altitude as the parallelogram is what part of the parallelogram? How then will you find the number of square units in a triangle when its base and altitude are given?

Express as a formula your answer to this question.

4. Find the number of square units in the following triangles: base 10 ft., altitude 8 ft.; altitude 12 ft., base 20 ft.; base 15 ft., altitude 30 ft.; altitude 18 ft., base 8 ft.; base 12 ft. 6 in.,

altitude 10 ft.; altitude 18 ft. 8 in., base 10 ft. 6 in.; base b ft., altitude r ft., base x ft., altitude r ft.

5. The gable of a house is 30 ft. wide and 15 ft. high. How many square feet in the two gables?

6. How many square feet in the sides, ends and two gables of a house 40 ft. long, 28 ft. wide, 20 ft. high if the gables are 12 ft. high?

LESSON XXI

REVIEW

1. Find the number of square feet in the surface of 15 steps if the height of the risers is 8 in., the width of the tread is 10 in., and the width of the steps is 5 ft.

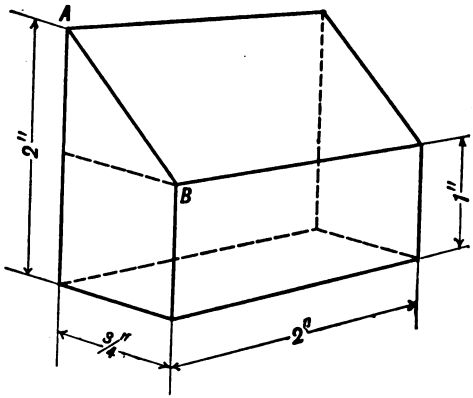


FIG. 25.

2. Find the number of square feet in this figure, scale 1/4 in. = 1 ft. 0 in. and AB = 5 ft.

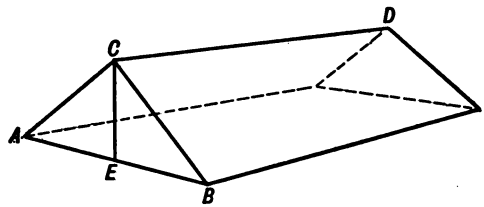


FIG. 26.

3. Find the number of square yards of canvas required to make a tent in the shape of the above figure if AB = 16 ft.,

CE = 6 ft. and CD = 20 ft., CB = 10 ft., allowing for waste and seams one-fourth.

4. How much siding will be necessary to side a house 32 ft. × 40 ft. × 18 ft. high if there are 20 windows 3 ft. × 7 ft. and 4 door openings 3 ft. 6 in. × 7 ft. long allowing one-fourth of the net area for waste?

5. A table top is 6 ft. $4\frac{1}{2}$ in. long, 3 ft. 2 in. wide and $1\frac{1}{2}$ in. thick. How much surface has it, not including underside?

6. How many shingles will be needed to cover a pitch roof 40 ft. long, with rafters 14 ft. in length, allowing 1000 shingles for each 100 sq. ft. of surface?

7. What is the cost of labor upon a $2000 house, if the material cost $896, excavation and cellar $84 and painting $55?

8. If r = 16, h = 33 find the value of $\pi r^2 h$.

9. If the scale is 1 in. = 3 ft. 6 in., what length on the drawing to represent 12 ft. 5 in.? 9 ft. 7 in.? 8 ft. 4 in.?

10. If 5.2 is taken for the square root of 27 show that the error is less than .004.

LESSON XXII

REVIEW

1. Find the side of a square that has the same area as a parallelogram 15 ft. × 12 ft. 6 in. Give result correct to three decimal places.

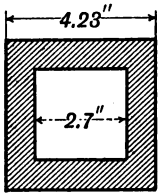


FIG. 27.

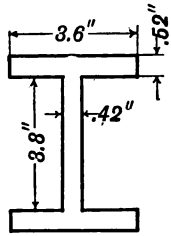


FIG. 28.

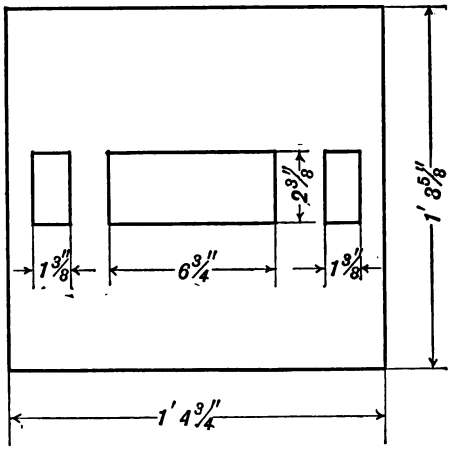


FIG. 29.

2. How many rafters are required for a pitch roof 40 ft. long if the rafters are set 2 ft. to centers?

3. Give the decimal part of an inch of each of the following micrometer readings:

Four large and 2 small of the barrel and 8 of the thimble; 6 large, 1 small of the barrel and 18 of the thimble; 2 large barrel and 3 of the thimble; 2 small barrel and 2 thimble.

4. What is the pitch and the lead of a screw with 15 threads to the inch? What is the micrometer reading of each?

5. The base of a triangle is 28 ft. 8 in. What is its altitude if it has the same area as a rectangle 10 ft. 3 in. × 24 ft. 6 in.?

6. Find the area of the shaded part of Fig. 27; also of the I section (Fig. 28).

7. Find the area of the part of the rectangle not included within the small rectangles of Fig. 29.

8. Compute the area of the following figure; c is perpendicular to a and b, and a and b are parallel:

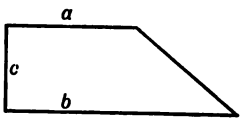


FIG. 30.

a = 10 ft. 3 in.; b = 15 ft. 6 in.; c = 4 ft. 5 in.

9. Find the side of the square that has the same area as the I section of problem 6; also the figure of problem 8.

LESSON XXIII

CIRCLES

A circle is a plane figure such that all the straight lines from a point to a closed line are equal.

The point is called the center of the circle.

The closed line is called the circumference of the circle.

Any one of the straight lines from the center to the circumference is called a radius (plural radii).

A straight line between any two points of the circumference is called a chord.

A chord through the center is called the diameter of the circle.

Any part of the circumference between two of its points is called an arc.

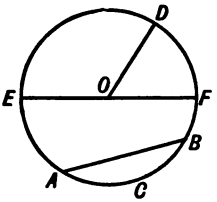


FIG. 31.

Name the center of the above circle. A radius, a chord, an arc, a diameter.

Is a diameter a chord? Are all chords diameters? Are any chords diameters?

A radius is what part of the diameter of the circle?

If the radius is 5.5 in., what is the diameter? If the diameter of a circle is 18, what is the radius?

Exercises

1. It is proved in geometry that *the circumference of a circle is equal to twice its radius multiplied by* π *or* $C = 2\pi R$.

Note on π. The number of times C contains D (diameter) is designated by the Greek letter π. This value cannot be exactly expressed by means of the digits, but correct to 15 decimal places:

$$\pi = 3.141592653589793$$

In actual work the value we shall give π depends upon the degree of accuracy needed. In some work $\pi = 3$ will give a sufficiently accurate result, in other work π may equal $3\frac{1}{7}$, etc. In most ordinary calculations $\pi = 3.1416$ is used.

2. *The number of square units in a circle is equal to the square of the radius multiplid by* π *or* $S = \pi R^2$.

3. Find the circumference of each of the following circles:

R = 15; 16; 12; 10; $9\frac{1}{2}$; $15\frac{1}{2}$; $12\frac{1}{3}$;

D = 24; 40; 16; 19; 31; 7; $8\frac{1}{2}$.

4. Find the number of square units in each of the circles in problem 3.

5. Find the number of square units in the ring between two concentric circles of radius 10 and 8 respectively.

6. A circular walk 4 ft. wide around a flower bed 20 ft. in diameter. Find the number of square feet in the walk.

7. Inscribed within a square is a circle whose diameter is 10 ft. Find the number of square feet between the circle and the sides of the square.

8. If the inner side of a circular running track is $\frac{1}{2}$ mile and the track is 20 ft. wide what is the length of the outer side of the track?

9. If the diameter of a circle is increased by 1 ft., how much is the circumference increased?

10. The diameter of the earth is about 8000 miles at the equator. Suppose an iron band lying everywhere upon the equator were stretched so that it would be everywhere $\frac{1}{2}$ ft. from the surface of the earth, how many feet would its length be increased?

11. If the radius of a circle were multiplied by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9, by 10, by what number would its circumference be multiplied? Its area?

12. What is the number of square feet in the surface of the track in problem 9?

LESSON XXIV

SPEED

Speed is rate of change of position. Speed is measured by a number of units of length per unit of time. For example, a body moving through 120 ft. each minute has a speed of 120 ft. per min. or 2 ft. per sec., or 7200 ft. per hr.

Exercises

1. A point is moving at a speed of 1 mile per min. What is its speed per hour? Per second?

2. What is the speed per minute of a train running 60 miles per hr.? 58 miles per hr.? 63 miles per hr.?

3. If the circumference of a wheel is 31.4 ft., what is the speed of a point on its rim, if the wheel is making 100 R.P.M. (revolutions per minute)?

4. What is the speed of a point on the rim of a cast-iron flywheel 1 ft. 0 in. in diameter, when the wheel is making 1680 R.P.M.?

5. A lathe spindle is running 1500 R.P.M. What is the speed of a point on the surface of a 6-in. cylinder placed in the chuck?

6. Ordinarily the maximum safe rim speed of cast-iron flywheel with solid rim is about 85 ft. per sec. Determine if any of the following are exceeding the safe speed:

1 ft. diam. making 2000 R.P.M.; 15 ft. diam. making 90 R.P.M.; 6 ft. diam. making 280 R.P.M.; 7½ ft. diam. making 220 R.P.M.; 9 ft. diam. making 185 R.P.M.; 8½ ft. diam. making 200 R.P.M.

7. Speeds for grindstones:

Machinist's tools, 800–1000 ft. per min.

Carpenter's tools, 500–600 ft. per min.

The maximum safe speed for a grindstone is ordinarily about 3400 ft. per min.

For grinding machinist's tools how many R.P.M. should a grindstone 3 ft. in diameter make? For carpenter's tools? What is the maximum safe number R.P.M.?

8. If S = surface speed of a point on a, the circumference of a revolving wheel or cylinder, D = the diameter of the wheel or cylinder and R.P.M. = number of revolutions per minute of the wheel or cylinder, express the formula for S in terms of D and R.P.M.

9. Use the formula derived in problem 8 and find S when

D = 10 and R.P.M. = 20; D = 5; R.P.M. = 100; D = 8; R.P.M. = 1200; D = 20; R.P.M. = 950.

10. If S = 500, D = 5; find R.P.M.; if S = 1000, R.P.M. = 500, find D; if S = 2500, R.P.M. = 1500, find C; if S = 3000, R.P.M. = 1000, find C; if S = 4000, C = 30, find R.P.M.; if S = 5000, C = 40, find R.P.M.

LESSON XXV

SPEEDS OF PULLEYS AND GEARS

If two pulleys are connected by a belt and one of the pulleys set in motion, the belt will cause the other pulley to move also, that is, motion is transmitted from one pulley to the other by means of the belt. The pulley that transmits motion to the belt is called the *driving pulley.* The pulley that is set in motion, or to which motion is transmitted, by the belt is called the *driven pulley.*

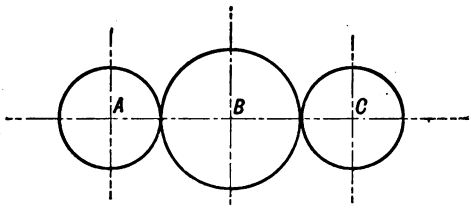


FIG. 32.

Motion is also transmitted by means of *gears.* In this case the motion is transmitted by direct contact, the teeth of the driving gear being made to mesh with the teeth of the driven. Should it, however, happen that the two gears are too far apart to mesh, an intermediate gear is used to transmit motion from the driving gear to the driven gear. Three or more gears meshing together form a train of gears, one or more of which is always an intermediate. Gear B is the intermediate.

Speed of pulleys or gears means the number of revolutions per minute (R.P.M.) the pulleys or the gears are making.

Exercises

1. If the driving gear has 24 teeth and the driven gear 8, when the driving gear has made one revolution, how many will the driven gear have made?

2. Driving gear 36 teeth, driven gear 9 teeth; driving gear 40 teeth, driven gear 5 teeth, driving gear 18 teeth, driven gear 7 teeth; driving gear 21 teeth, driven gear 6 teeth.

How many revolutions will the driven gear in each of the above make for one revolution of the driving gear?

3. If the driving gear contains 32 teeth and is making 40 R.P.M. what is the speed of the driven gear with 9 teeth?

4. The front sprocket of a bicycle contains 24 teeth and the rear sprocket 8 teeth, how many revolutions will the pedals make in going 1 mile? The wheels of the bicycle being 28 in. in diameter?

5. The driving gear has 30 teeth and the driven gear 10 teeth. If they are connected with an intermediate of 40 teeth, what number of revolutions does the driven gear make for each of the driving gear? Explain why this is true. What effect upon the speed of the driven gear has the intermediate?

6. The diameter of the driving pulley is 12 in. and its speed 300 R.P.M. What is the speed of the driven pulley whose diameter is 4 in.; 3 in.; 5 in.?

7. The diameter of the driving pulley is 10 in. and its speed 900 R.P.M. Required the speed of the driven pulley of diameter 4 in.

8. On the driving shaft is a 24-in. pulley making 300 R.P.M. What is the speed of the driven shaft which has a 10-in. pulley belted to the driver?

9. Find the surface speed of each pulley in problem 8.

10. If the surface speed of 6-in. pulley is 3000 ft. per min.; how many R.P.M. is it making?

11. The maximum safe surface speed of a grindstone is

2400 ft. per min. Find the maximum safe number R.P.M. a 6-ft. stone may make.

12. If the surface speed of an emery wheel making 600 R.P.M. is 4000 ft., what is its diameter?

13. A driven pulley 5 in. in diameter has a speed of 2500 R.P.M. If the speed of the driving pulley is 500 R.P.M., what is its diameter?

14. The speed of the driving pulley is N = R.P.M. and its diameter D. Find the formula for the speed S of the driven pulley whose diameter is d.

15. Use formula of problem 14 and find S if N = 120, D = 12, and d = 6; if N = 1200, D = 20, d = 5, find S; if S = 500, d = 10, D = 25, find N; if S = 2500, d = 8, D = 24, find N; if N = 3000, D = 18, d = 5, find S; if N= 1800, D = 40, d = 8, find S; if S = 900, D = 20, d = 2, find N.

16. If D = number of teeth in driving gear, d = in driven gear N = R.P.M. of D and n = R.P.M. of d, show that

$$\frac{D}{d} = \frac{n}{N}$$

17. Use formula of problem 16 to find D when d = 10, n = 30, N = 60; to find N if D = 40, d = 24, n = 12; to find d if D = 80, N = 4, n = 8; to find D if N = 12, n = 6 and d = 20; to find N if D = 80, d = 28 and n = 2.

18.

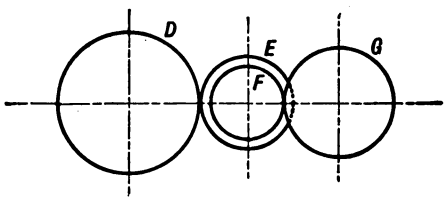


FIG. 33.

In the above train, E and F are keyed to the same shaft. D has 60 teeth, E 40, F 30 and G 45. Find the number of revolutions of G for each revolution of D.

Suppose an intermediate of 60 teeth is placed between F and G, then what is the result?

19. Let D = 100, E = 90, F = 70 and G = 55. If R.P.M. of D = 40 find R.P.M. of G.

20.

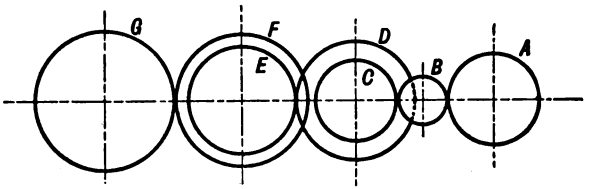


FIG. 34.

E and F are keyed to the same shaft, likewise C and D. A = 40, B = 20, C = 35, D = 50, E = 45, F = 55 and G = 60. For each revolution of A, find number of revolutions of G.

21. Let G = 80, F = 70, E = 60, C = 50, D = 55, B = 20 and A = 40. When G has made one revolution, how many has A made?

LESSON XXVI

CUTTING SPEED AND FEED

When turning or cutting a cylindrical piece in a lathe the number of lineal feet of the surface of the piece cut by the tool in 1 min. is called *cutting speed* of the tool. For example, if the piece is making 20 R.P.M. and its circumference is 4 in., the cutting speed is evidently 4 in. $\times$ 20 = 80 in. = $6\frac{2}{3}$ ft. per min.

The distance the tool advances along the work in each revolution is called *feed.* It is *expressed* in ordinary work as a fractional part of an inch. For example, if the tool advances $\frac{1}{16}$ in. along the work in one revolution the feed is $\frac{1}{16}$ in. The width of the chip is equal to the feed.

Exercises

1. Is the surface speed per minute of a revolving cylinder the same as the cutting speed of the tool? Why?

2. A piece of round stock 2 in. in diameter is making 40 R.P.M. What is the cutting speed?

3. If the feed is $\frac{1}{16}$ in. and the work has a speed of 42 R.P.M. how long will it require to turn a piece 3 in. long?

4. A piece of work revolves 50 times while the tool advances $1\frac{9}{16}$". Find the feed.

5. If the feed is $\frac{1}{32}$ in. and the cutting speed 20 ft. per min., find the time required for the tool to travel 1 in. along a piece 4 in. in diameter.

6. When cutting soft steel the speed may be 100 ft. per min. if the depth of the cut is $\frac{1}{2}$ in. and the feed $\frac{1}{32}$ in. Find the

R.P.M. a cylinder of soft steel 3 in. in diameter is making from the above data.

7. If the feed per revolution of a drill is .0075 in., find the time required to drill a hole .25 in. in diameter through a rectangular piece of steel 1¼ in. thick, if the drill is making 735 R.P.M.

8. How many strokes of the shaper will be required to rough cut a rectangular piece 2⅛ in. wide, if the feed is 1/32 in.?

9. A round piece 36 in. long and 2 in. in diameter is to be turned to a diameter of 1⅞ in. How long will be required to do the work if the feed of the rough cut is 1/40 in. and the finish cut 1/60 in. and the work has a speed of 120 R.P.M.

10. If F = feed, N = R.P.M. and D = distance the tool moves find F in terms of D and N.

11. Use the formula of problem 10.

If F = 1/10 in., R.P.M. = 50, find D; if D = 2, F = 1/32 in., find R.P.M.; if R.P.M. = 40, F = ⅛ in., find D; if D = 1, R.P.M. = 32, find F; if R.P.M. = 32, D = 2, find F; if D = 3, R.P.M. = 32, find F.

LESSON XXVII

AREA OF SURFACE OF CYLINDER

In geometry it is proved that *the area of the curved surface of a right cylinder is the circumference of its base multiplied by its altitude.*

The complete area includes the area of the curved surface and the area of the two ends.

Exercises

1. Find the number of square units in the curved surface of each of the following cylinders:

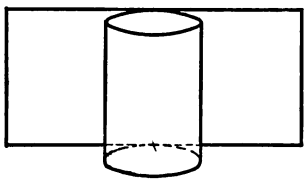


FIG. 35.—Figure of a cylinder showing its surface changed to a rectangle.

Circumference of base 28 in., altitude 10 in.; circumference of base 38 ft., altitude 12 ft.; circumference of base 42.5 ft., altitude 22 ft.; circumference of base 15 ft. 8 in., altitude 9 ft.

2. Find the number of square units in the curved surface of each of the following cylinders:

Altitude 18 ft., radius of base 10 ft.; altitude 10 ft., radius of base 8 ft.; altitude 8 ft., radius of base 12 ft.; altitude 15 ft., radius of base 9 ft. 6 in.

3. Find the number of square units in the complete surface of each of the cylinders in problem 2.

4. Find the cost of painting a cylindrical column 20 ft. high and 3 ft. in diameter at 5¢. per sq. ft.

5. How many square feet of tin will be required to line the bottom and side of a cylindrical tank 10 ft. high and 8 ft. in diameter? What will be the cost at 10¢. per sq. ft.?

6. Express the formula for area of the curved surface of a cylinder; the complete area of a cylinder; if H = its altitude and R = radius of its base.

LESSON XXVIII

REVIEW

1. Find the side of a square that has the same area as the complete area of a cylinder 2 ft. in diameter and 4 ft. long. $\pi = 3.1416$.

2. A shaft to which are attached two pulleys is making 300 R.P.M. Compare the surface speeds of the two pulleys if their diameters are respectfully 18 in. and 10 in.

3. The outside diameter of a pipe is 2 in. and the inside diameter $1\frac{3}{4}$ in. Find the difference between the outside and inside areas of a piece 10 ft. long.

4. A grinding wheel 6 in. in diameter has a surface speed of 500 ft. per min. The wheel is attached to a shaft which has a 3-in. pulley, which is belted to 10-in. pulley on another shaft on which is also a 4-in. pulley; this pulley is belted to a 12-in. pulley on another shaft on which is another 12-in. pulley which is belted to a 5-in. pulley attached to a shaft run directly by the motor. Find R.P.M. of motor.

5. Three pipes respectively 3 in., 4 in. and 6 in. in diameter are discharging into a header. What is the diameter of the header, if the rate of flow in all four pipes is the same and all the pipes are full?

6. How many R.P.M. must a cylinder 10 in. in diameter make in order that the tool may advance 2 in. in 3 min., if the feed is $\frac{3}{64}$ in.?

7. If the driving gear has 75 teeth and a speed of a 40 R.P.M. what is the speed of the driven gear with 25 teeth?

8. How will you set the micrometer for .149 in.? .403 in.? .738 in.?

9. How many revolutions must a thread whose pitch is $\frac{1}{20}$ make to advance .2 in.? $\frac{1}{2}$ in.?

10. If B = 36, b = 49 and H = 27, find V.

$$V = \frac{H}{3}(B + b + \sqrt{Bb}).$$

11. If R = 12, r = 9 and H = 15, find V.

$$V = \frac{\pi H}{3}(R^2 + r^2 + Rr).$$

LESSON XXIX

VOLUME OF A PRISM

1. A prism is a solid whose bases are parallel and whose faces are parallelograms.

2. A prism takes its name from the shape of its base, *e.g.*, if the base is a triangle, it is called a triangular prism; if a quadrilateral, a quadrangular prism, etc.

3. The perpendicular between the bases is called the altitude of the prism.

4. The intersections of the faces are called the edges of the prism.

5. If the edges are perpendicular to the bases the prism is a right prism. We will consider only right prisms. The edge of a right prism is equal to the altitude of the prism.

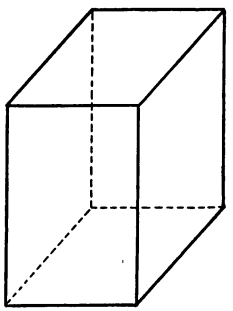


FIG. 36.—Right Prism.

6. A prism whose bases and faces are squares is called a cube.

7. Measuring a solid consists in finding the number of standard cubes a solid contains.

A standard cube is a cube each of whose edges is a standard unit of length; *e.g.*, a cubic inch, cubic foot, etc., is a cube each of whose edges is an inch, a foot, etc.

8. The number of cubes a solid contains is called its volume in terms of the cube; *e.g.*, if a solid contains 15 cu. ft., its volume is 15 in terms of a cubic foot.

9. Find the volume of a prism 6 in. long, 3 in. wide and 3 in. high. How many cubes in a section along the base? How many in one layer? How many in three layers?

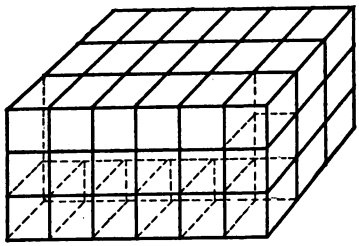


FIG. 37.

10. Find the number of cubes in each of the following prisms:

8 in. × 3 in. × 4 in.; 15 in. × 12 in. × 6 in.; 9 ft. × 9 ft. × 9 ft.; 7 ft. × 7 ft. × 10 ft.; 18 ft. × 20 ft. × 8 ft.; 17 ft. × 6 ft. × 4 ft.; 10 ft. × 8 ft. × 12 ft.; 9 ft. × 8 ft. × 6 ft.; 25 ft. × 15 ft. × 20 ft.; 10 ft. 6 in. × 8 ft. 6 in. × 7 ft. 3 in.; 15 ft. 8 in. × 12 ft. 9 in. × 10 ft. 6 in.

11. If each edge of a cube is 5 in. how many cubic inches in it? If 6 in.? If 10 in.? If 12 in.? If 15 in.? If a in.? If b in.? If c in.? If n in.?

12. How many cubic inches in a cubic foot? How many cubic feet in a cubic yard?

Fill in the following table and learn it.

——cu. in. = 1 cu. ft.

——cu. ft. = 1 cu. yd.

13. Change to cubic inches, 18 cu. ft.; 25 cu. ft.; 33⅓ cu. ft.; 12½ cu. ft.; 100 cu. ft.

14. Change to cubic yards, 5760 cu. ft.; 9000 cu. ft.; 1765 cu. ft.

15. Change to cubic feet, 15,625 cu. in.; 172,800 cu. in.; 2456 cu. in.

16. The dimensions of a rectangular prism are a, b and c, derive the formula for its volume.

17. If $a = 4$, $b = 5$, $c = 3$, $V = ?$
$a = 10$, $b = 9$, $c = 8$, $V = ?$
$a = 8$, $b = 6$, $c = 12$, $V = ?$

18. If $V = 144$, $a = 4$, $b = 9$, $c = ?$
$V = 250$, $a = 6$, $b = 10$, $c = ?$
$V = 400$, $b = 5$, $c = 12$, $a = ?$
$V = 920$, $a = 9$, $c = 10$, $b = ?$

LESSON XXX

REVIEW

1. Find the number of cubic yards of earth taken from a cellar 27 ft. long, 25 ft. wide and 5 ft. deep.

2. A ditch 1 mile long, 10 ft. wide and 6 ft. deep had how many cubic yards of earth removed from it?

3. How many cubic feet of masonry in a wall 40 ft. long, 15 ft. high and 4 ft. 6 in. wide?

4. A wall 2 ft. thick and 8 ft. high is built around a lot 50 ft. $\times$ 150 ft. Find the number of cubic yards of masonry in it if there is an opening 10 ft. wide in the wall. (Two solutions.)

5. If your schoolroom is 12 ft. high, how many cubic feet of air are in the room?

6. A piece of lumber 16 ft. and 2 in. $\times$ 8 in. contains how many cubic feet of wood?

7. What will be the cost of a house 36 ft. $\times$ 25 ft. and 20 ft. high at 20¢. per cu. ft.?

8. A tank 10 ft. square and 12 ft. deep will hold how many gallons of water? 1 cu. ft. contains $7\frac{1}{2}$ gal.

9. How many cubic feet of wood in a pile 8 ft. long, 4 ft. wide and 4 ft. high?

10. Find the number of cubic feet in a box 5 ft. 6 in. long, 3 ft. 8 in. wide and 2 ft. 9 in. high.

11. Use the formula for the volume of a prism and solve for a in terms of the other letters, likewise for b, for c.

12. State in words the results you have obtained in problem 11.

LESSON XXXI

REVIEW

1. A piece of steel $\frac{3}{4}$ in. square and 7.25 in. long is taken to make a lathe tool. Find its weight if 1 cu. in. of steel weighs .28 lb.

2. If the piece in problem 1 weighs 4 lb., what was its length?

3. The base of a rectangular prism is 8 ft. $\times$ 5 ft. What is its height, if it has the same volume as a 6-in. cube?

4. A cube of steel 6-in edge is hammered when hot into a rectangular prism whose base is 5.6 in. $\times$ 3.4 in. Find the length of the prism.

5. A rectangular piece of steel 8 in. $\times$ 5 in. $\times$ 3.5 in. has each dimension reduced $\frac{1}{8}$ in. How much is its weight reduced?

6. A liquid gallon contains 231 cu. in. A rectangular measure 4 in. square must be how deep to hold 1 qt.? 1 gal.?

7. If the I section of problem 6, lesson 22, is 8.25 ft. long, what is its volume?

8. A rectangular tank 15 ft. long and 12 ft. wide contains 500 gal. of water. How deep is the water in feet?

9. If the weight of the water in the tank of problem 8 weighs 1 ton, how deep is the water? 1 cu. ft. weighs 62.5 lb.

10. Two rectangular pieces of metal 5 ft. $\times$ 5 ft. $\times$ 8 ft. and 6.5 ft. $\times$ 3.4 ft. $\times$ 10 ft. are melted and cast into a single rectangular piece 8 ft. square. How long is the new piece?

11. A bar of iron 2 in. square and 1 in. long is drawn out until it is $1\frac{1}{2}$ in. $\times$ 1 in. What is its length?

12. If a piece of stock $2\frac{1}{2}$ in. square and 8 in. long is forged into a piece 2 in. square, how long is the new piece?

13. The sides and bottom of an open steel tank 4 ft. square and 6 ft. high outside dimensions are 2 in. thick, find the weight of the tank.

14. A piece of steel 14.5 in. $\times$ 16 in. and 2 in. thick has a rectangular hole 8 in. $\times$ 4.25 in. cut in it. What is the weight of the piece of steel?

15. If the scale is $1'' = 3'\ 4''$ how long must the drawing be for each of the following lengths:

12 ft. 0 in; 15 ft. 7 in; 16 ft. 9 in.; 7 ft. 10 in.; 23 ft. 8 in.?

LESSON XXXII

REVIEW OF PERCENTAGE

The expression 6% means .06 or $\frac{6}{100}$.

The expression 28.3% means .283 or $\frac{283}{1000}$.

25% = $\frac{25}{100}$ = $\frac{1}{4}$; 33$\frac{1}{3}$% = $\frac{33\frac{1}{3}}{100}$ = $\frac{1}{3}$; 20% = ?/100 = ?; 50% = ? = ?

Exercises

1. Express as a decimal fraction without their denominators each of the following:

12$\frac{1}{2}$%; 32%; 35.3%; 96.4%; 76.5%; 87.7%.

2. 25% of 144 = ? 12$\frac{1}{2}$% of 840 = ? 20% of 255 = ? 33$\frac{1}{3}$% of 175 = ? 75% of 164 = ? 66$\frac{2}{3}$% of 930 = ?

3. 28% of what number is 28?

4. 73% of what number is 146?

5. 86.2% of what number is 2586?

6. 28 is what per cent. of 560? 35 is what per cent. of 70?

7. 144 is what per cent. of 770? 29 is what per cent. of 125?

8. How many board feet of flooring must be purchased for a floor 25 ft. × 18 ft. if an allowance of 20% is added for waste?

9. In finishing a piece of steel its weight was reduced 1.5%. What was the weight of the finished piece if the rough piece weighed 78.31 lb.?

10. How many pounds of lead are there in 168 lb. of soft solder, if 33$\frac{1}{3}$% the solder is lead?

11. A certain grade of steel contains 3.4% nickel. How many pounds of nickel in 1 ton of this grade of steel?

12. The foot shrink rule used by the pattern maker is 12$\frac{1}{8}$ in. in length. The carpenter's foot rule is what per cent. of the length of the pattern maker's foot rule?

13. About .37% of soft steel is silicon. How much silicon is there in 1500 lb. of soft steel?

LESSON XXXIII

BOARD MEASURE

1. A board foot is a piece of lumber having an area of 1 sq. ft. on its flat surface and thickness of 1 in. or less. A board foot 1 in. thick contains how many cubic inches?

2. In estimating the number of board feet in a piece of wood it is customary to estimate thickness less than 1 in. as an entire inch; *e.g.*, a board 12 ft. long, 8 in. wide and $\frac{7}{8}$ in. thick is estimated the same as if it were 12 ft. long, 8 in. wide and 1 in. thick.

3. Find the number of board feet in a piece of lumber $1\frac{1}{2}$ in. thick, 8 in. wide and 16 ft. long.

4. Find the number of board feet in a piece of lumber 6 in. thick, 8 in. wide and 16 ft. long.

5. Find the number of board feet of lumber in each of the following:

1 in. × 4 in. × 12 ft.; $1\frac{1}{2}$ in. × 8 in. × 16 ft.; 5 in. × 4 in. × 18 ft.; 2 in. × 6 in. × 16 ft.; $\frac{7}{8}$ in. × 6 in. × 20 ft.; 2 in. × 9 in. × 16 ft.; 7 in. × 18 in. × 20 ft.; 7 in. × 8 in. × 12 ft.; $1\frac{1}{2}$ in. × 6 in. × 12 ft.; $\frac{1}{2}$ in. × 8 in. × 16 ft.; $1\frac{1}{2}$ in. × 10 in. × 18 ft.; 4 in. × 6 in. × 14 ft.

6. How many board feet of flooring 1 in. thick are required for a floor 20 ft. 6 in. × 15 ft. 9 in. if 25% is allowed for waste?

7. How many board feet in 10 pieces of lumber 2 in. × 8 in. and 16 ft. long.

LESSON XXXIV

REVIEW

1. The outside dimensions of a box are 4 ft. × 3 ft. × 2 ft. If the boards are 1 in. thick, how many board feet are in the box and its lid?

2. The inside dimensions of a coal bin are 10 ft. × 6 ft. × 5 ft. If the sides are 2 in. thick, how many board feet of lumber are in its sides, if one of the sides is cellar wall?

3. The floor of a veranda is laid of boards $1\frac{1}{2}$ in. thick. If the floor is 8 ft. × 23 ft. 6 in., how many board feet of flooring does it require, allowing 10% for waste. Find the cost at $60 per M.

4. How many board feet of lumber will be required to floor a platform 16 ft. 4 in. long × 11 ft. 6 in. wide, the lumber to be 1 in. thick, if no allowance is made for waste? If 10% is allowed for waste?

5. Find the area (complete) of a cylinder 10 ft. high and 10 ft. in diameter.

6. How many square feet of tin will be required to make a pipe 4 ft. long and 7 in. in diameter if 1 in. is allowed for the seam? Find the cost at 10¢. per sq. ft. $\pi = 3\frac{1}{7}$.

7. If a wheel 2 in. in diameter is making 2500 R.P.M., through how many feet will a point on its rim pass in 1 min.? In 1 hr.?

8. A chimney 30 ft. high is 18 in. square and has a flue 12 in. square. How many cubic feet in the masonry of the chimney?

9. If the feed is $\frac{1}{8}$ in. and the work has a speed of 40 R.P.M. how long will it require to turn a piece $4\frac{1}{4}$ in. long?

10. If the feed of the shaper is $\frac{1}{32}$ in., how many strokes must it make to rough cut a piece $3\frac{1}{16}$ in wide?

LESSON XXXV

WOODWORKING PROBLEMS

Rough undressed lumber for shopwork is sawed into the following standard thickness:

1 in.; 1½ in.; 1¼ in.; 2 in.; 2½ in.; 5 in.; 3½ in.; and 4 in.

In dressing lumber about ⅛ in. of its thickness is removed, that is a 1-in. board when dressed will be ⅞ in. thick.

If a dressed board 1¾ in. in thickness is wanted a 2-in. thickness must be dressed down ¼ in.

In ordering lumber from the mill the number of pieces wanted, the thickness, the width and length of each piece must be given.

A piece of lumber 2 in. × 2½ in. × 40 in. means the thickness is 2 in., the width 2½ in. and the length 40 in.

A mill bill is an order on the mill for lumber wanted. It must contain the number of pieces, kind of wood and dimensions of each piece. The dimensions are usually given for rough lumber.

In estimating the amount of lumber required for a given job, add about 25% of the actual number of board feet in the finished article. This allowance is for waste. Waste in handling lumber results from any or all of the following causes:

1. Saw cuts about ⅛ in.
2. Checks or cracks in lumber especially at ends.
3. Knots.
4. Small pieces resulting from sawing.

5. General defects in lumber.
6. Dressing of lumber.

Exercises

1. Find the total number of board feet and the cost of the following mill bill for a type stand. Allow 25% for waste.

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
6 legs	Chestnut	$2\frac{3}{8}'' \times 2\frac{3}{8}'' \times 43''$		7¢.	
2 rails	Chestnut	$1\frac{1}{2}'' \times 3\frac{1}{8}'' \times 72''$		7¢.	
2 rails	Chestnut	$1\frac{1}{2}'' \times 2\frac{5}{8}'' \times 72''$		7¢.	
3 rails	Chestnut	$1\frac{1}{2}'' \times 3\frac{1}{8}'' \times 22''$		7¢.	
3 rails	Chestnut	$1\frac{1}{2}'' \times 2\frac{5}{8}'' \times 22''$		7¢.	
1 top	Chestnut	$1'' \times 2\frac{3}{8}'' \times 75''$		7¢.	
		Total.....			
		Allowance.....			
		Grand total.....			

2. Find totals for the following used in making step ladders.

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
2	Chestnut	$\frac{7}{8}'' \times 3\frac{5}{8}'' \times 90''$		7¢.	
2	Chestnut	$\frac{7}{8}'' \times 2\frac{1}{4}'' \times 75''$		7¢.	
3	Chestnut	$\frac{7}{8}'' \times 2\frac{1}{4}'' \times 36''$		7¢.	
6	Chestnut	$\frac{7}{8}'' \times 4\frac{1}{2}'' \times 20''$		7¢.	
1	Chestnut	$\frac{7}{8}'' \times 9\frac{1}{4}'' \times 22\frac{1}{2}''$		7¢.	

3. Find totals for the following used in making case racks for print shop.

The waste in this problem was due to an error in cutting and the pieces had to be replaced.

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
24	Chestnut	$2\frac{3}{8}'' \times 2\frac{3}{8}'' \times 43''$		7¢.	
12	Chestnut	$1\frac{1}{4}'' \times 3\frac{1}{8}'' \times 72''$		7¢.	
12	Chestnut	$1\frac{1}{4}'' \times 2\frac{5}{8}'' \times 72''$		7¢.	
18	Chestnut	$1\frac{1}{4}'' \times 3\frac{1}{8}'' \times 22''$		7¢.	
18	Chestnut	$1\frac{1}{4}'' \times 2\frac{5}{8}'' \times 22''$		7¢.	
6	Chestnut	$\frac{7}{8}'' \times 2\frac{3}{8}'' \times 75''$		7¢.	
waste 4	Chestnut	$2\frac{3}{8}'' \times 2\frac{3}{8}'' \times 41''$		7¢.	
1	Chestnut	$1\frac{1}{4}'' \times 2\frac{5}{8}'' \times 72''$		7¢.	
1	Chestnut	$1\frac{1}{4}'' \times 3\frac{1}{8}'' \times 72''$		7¢.	

LESSON XXXVI

VOLUME OF A CYLINDER

1. *In geometry it is proved that the volume of a cylinder is equal to the area of its base multiplied by its altitude.*

2. Find the volume of each of the following cylinders:

Altitude 15 ft., radius of base 7 ft.; altitude 10 ft. radius of base 8 in.; altitude 18 ft., radius of base 3 ft.; altitude $18\frac{1}{2}$ ft., radius of base 1 ft.; altitude 25 ft., diameter of base 10 ft.; altitude 16 ft. 8 in., diameter of base 3 ft.

3. Find the number of gallons a cylindrical tank 10 ft. high and 8 ft. in diameter will hold. 1 cu. ft. = $7\frac{1}{2}$ gal.

4. At 25¢. per cu. ft. what will be the cost of a stone column 15 ft. high and 2 ft. in diameter?

5. A cylindrical pipe 25 ft. long and 6 in. in diameter, inside dimensions, has water running through it at the rate of 25 ft. per min. How many gallons will pass through it in 1 min.? In 1 hr.?

6. A cylinder 4 ft. in diameter and 10 ft. high contains how many times as much volume as a cylinder 2 ft. in diameter and 10 ft. high?

7. A prism 4 in. square and 2 ft. long must have how many cubic inches cut from it to give a cylinder 4 in. in diameter?

8. Find the volume of a cylinder the radius of whose base is R and whose altitude is H.

9. Use the formula of problem 8 and solve for H in terms of V, π and R; for R in terms of V, π and H.

10. If V = 100 π, R = 5, find H.
If V = 250 π, H = 10, find R.
If V = 400, R = 6, find H.
If V = 500, H = 10, find R.

LESSON XXXVII

REVIEW

1. How deep must a cylindrical cistern 6 ft. in diameter be to hold 500 gal.? 1200 gal.?

2. A dealer was using a cylindrical measure 6 in. in diameter and $7\frac{3}{4}$ in. deep. He called it a gallon. Was it? $\pi = 3.1416$.

3. A cylinder whose altitude is h, radius of base 2r, contains how many times as much volume as a cylinder altitude h and radius of base r?

4. A water main is 3 ft. internal diameter and 1 in. thick. Find the weight of 8 ft. 1 cu. in. of pipe weighs .26 lb.

5. A cylindrical tank 8 ft. internal diameter is full of water; if the surface of the water is lowered 3 ft., how many gallons were drawn from the tank?

6. A closed cylindrical can 8 in. deep and 8 in. in diameter is placed inside of a cubical box 12 in. each edge, inside diameter. How much water can be put into the box? $\pi = 3\frac{1}{7}$.

7. Which contains the more volume, a cylinder 6 ft. deep and 4 ft. in diameter or a cylinder 4 ft. deep and 6 ft. in diameter? The greater area?

8. How much air passes into a room through an 18-in. pipe each minute if air is flowing through the pipe 250 ft. per min.?

9. A piece of tin 24 in. long and 10 in. wide is rolled into a can; how many gallons will the can hold if $\frac{1}{2}$ in. is allowed for the seams? (Two solutions.) The top and bottom of can do not come out of the piece 24 in. $\times$ 10 in.

10. How many loads of earth must be removed to dig a cesspool 10 ft. deep and 10 ft. in diameter? 1 load equals 1 cu. yd.

11. A rotary air-fan delivers 2000 cu. ft. of air per min. through a pipe 2 in. in diameter. Find the rate of discharge in feet per minute.

LESSON XXXVIII

REVIEW

1. The following was received for use in the machine shops of this High School. Find its cost at 17¢. per lb. 1 cu. ft. of steel weighs 490 lb. $\pi = 3\frac{1}{7}$.

JESSOP'S TOOL STEEL (ROUND)

Size	Total length	Weight	Cost
$1\frac{5}{8}''$	8′5″	?	?
$1\frac{1}{2}''$	23′8″	?	?
$1\frac{3}{8}''$	7′5″	?	?
$1\frac{1}{8}''$	20′3″	?	?
$1\frac{1}{16}''$	16′5″	?	?
1″	9′3¾″	?	?
$\frac{3}{4}''$	7′7″	?	?

Total.

2. Find the weight of a piece of round steel 1 ft. long and 2 in. in diameter. Find its cost at 17¢. per lb.

3. Estimate the cost at $2\frac{1}{2}$¢. per lb. of the following bill of machine steel received at this High School:

$1\frac{3}{4}$ in. in diameter and 36 ft. long; $1\frac{5}{8}$ in. in diameter and 37 ft. long; $1\frac{1}{4}$ in. and $18\frac{1}{2}$ ft.; $1\frac{1}{8}$ in. and $37\frac{1}{2}$ ft.; 1 in. and $8\frac{1}{2}$ ft.; $\frac{7}{8}$ and $26\frac{1}{2}$ ft.

4. A piece of cold rolled soft steel 16 ft. long and 2 in. in diameter is worth how much at 5¢. per lb.?

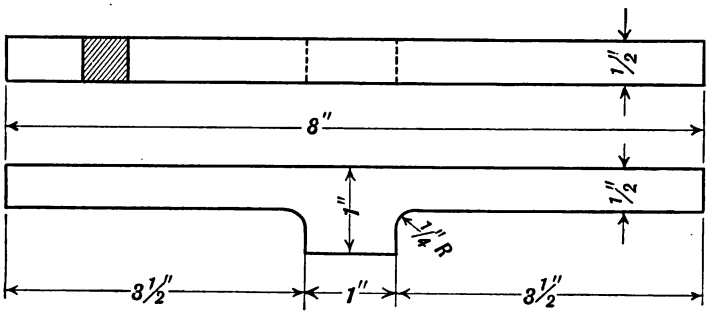
LESSON XXXIX

FORGE SHOP PROBLEMS

1. A shovel for the foundry was made from the following stock: $\frac{3}{4}$ in. $\times$ 2 in. $\times$ $6\frac{1}{2}$ in. for the blade and $\frac{7}{16}$ in. round $\times$ 21 in. for the handle. Find the weight of the shovel. 1 cu. ft. of iron weighs 480 lb.

2. Find the cost of 50 shovels of problem 1 at $3\frac{3}{4}$¢. per lb.

3. A piece of stock 1 in. $\times$ 1 in. $\times$ 15 in. was used to make one pair of tongs. Find the weight of 35 pairs of the tongs.



Stock ½" x 1"x 6"

FIG. 38.

4. A three-way piece was made from stock $\frac{1}{2}$ in. $\times$ 1 in. $\times$ 6 in. Find the cost of the piece at $3\frac{3}{4}$¢. per lb.

5. A clothes hook was made from a piece of stock $\frac{3}{8}$ in. in diameter and 6 in. long. Find the weight of 100 of these hooks.

6. Find the weight of the material used to make a three-way piece if the stock is $\frac{3}{8}$ in. in diameter and $4\frac{1}{2}$ in. long.

7. What is the weight of 100 hooks for foundry shovels if the stock is 10 in. × ½ in. in diameter?

8. In Fig. 38 compute the amount of stock in the shouldered piece after 3 in. are cut from each end.

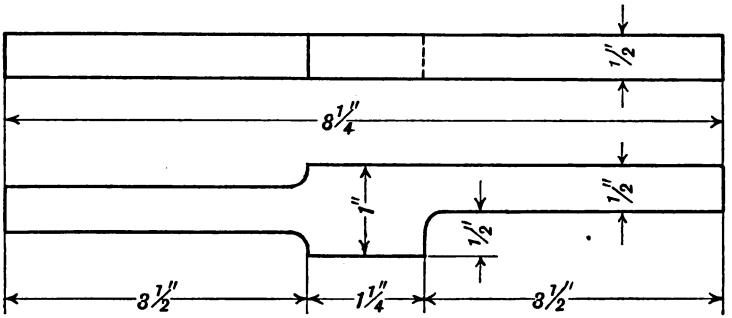


FIG. 39.

9. In Fig. 39 if 3½ in. are cut from each end of piece, find weight of shouldered piece.

LESSON XL

REVIEW

1. Show that $\sqrt{3} = 1.732$, also $\sqrt{2} = 1.414$. Memorize these numbers.

2. $15\sqrt{3} = ?$ $12\sqrt{2} = ?$ $25\sqrt{3} = ?$ $75\sqrt{3} = ?$ $12\frac{1}{2}\sqrt{2} = ?$ $16\sqrt{2} = ?$

3. $\dfrac{3\frac{1}{4} + 5\frac{1}{2} + 7\frac{1}{3} \times \frac{6}{7}}{1\frac{1}{2} + 3\frac{1}{4} + 6\frac{1}{2} + 4} = ?$

4. Find the volume of the cube the area of one of whose faces is 64 sq. in.

5. How many board feet in the cube in problem 4?

6. If a window frame cost \$1.65 what will be the cost of the window frames in a house with 15 window openings?

7. How many board feet of lumber 1 in. thick will be used in laying a floor 28 ft. × 24 ft. allowing 25% for waste?

8. A man paid \$35 for repainting a house, which was $\frac{1}{30}$ of the amount paid for the property. Find the cost. If he sold the property for \$1500 did he gain or lose and how much?

9. A cylindrical bucket is 18 in. in diameter and 2 ft. high. How many gallons will it hold?

10. The gable of a house is 12 ft. high and 24 ft. wide. How many shingles will be required for the two gables? Allow 750 shingles for each 100 sq. ft.

11. $\sqrt{157632} = ?$ $\sqrt{57.69} = ?$ $\sqrt{875.14} = ?$

12. An auger hole 1 in. in diameter is bored through a piece of wood 1 ft. long and 6 in. square. Find the volume remaining in the wood if the hole is lengthwise and perpendicular to the base?

13. Find the number of inches of 6-in. round stock required to make a forging $2\frac{1}{2}$ in. in diameter and 55 in. long.

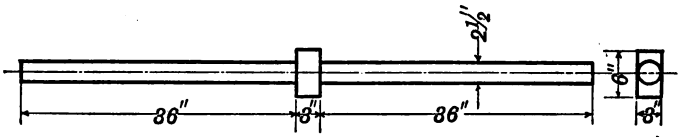


FIG. 40.

14. How long a piece of 6-in. round stock will be required to forge the above?

LESSON XLI

SIMPLE EQUATIONS

Find the value of the letter in each of the following:

1. $3x = 6$, $4x = 16$, $5x = 25$, $6x = 36$.

2. $7x = 14$, $5x = 30$, $7y = 14$, $3a = 15$.

3. $10b = 30$, $12c = 36$, $15k = 45$, $16p = 48$.

4. $5p = 36$, $7r = 15$, $8m = 30$, $5L = 36$.

5. $x^2 = 36$, $y^2 = 25$, $m^2 = 49$, $n^2 = 100$.

6. $2x^2 = 100$, $2x^2 = 50$, $3n^2 = 75$, $4s^2 = 400$.

7. $6x^2 = 216$, $5L^2 = 80$, $4p^2 = 64$, $10k^2 = 6250$.

8. $x^2 = 15$, $2y^2 = 30$, $3d^2 = 48$, $4m^2 = 72$.

9. $a + 5 = 10$, $m + 10 = 25$, $y + 10 = 24$.

10. $r + 6 = 36$, $d + 7 = 49$, $y + 8 = 50$.

11. $10d + 4 = 44$, $55s + 6 = 31$, $7e + 10 = 31$.

12. $8k + 3 = 27$, $10m + 5 = 45$, $15p + 10 = 55$.

13. $6x + 5 = 30$, $7y + 4 = 28$, $10r + 6 = 40$.

14. $x^2 + 9 = 25$, $x^2 + 25 = 169$, $y^2 + 36 = 100$.

15. $d^2 + 36 = 100$, $y^2 + 36 = 100$, $g^2 + 100 = 674$.

16. $25 + a^2 = 169$, $144 + d^2 = 169$, $p^2 + 64 = 81$.

17. If three times a number is increased by 5, the sum is 14. What is the number?

18. Seven times a number and 12 more is 26. What is the number?

19. What number increased by five times itself will give 30?

20. If John's money were multiplied by 5 and $15 added to the product the result would be $100. How much money has he?

LESSON XLII

THEOREM OF PYTHAGORAS

It is proved in geometry that

"*The square of the hypothenuse of a right triangle is equal to the sum of the squares of its legs.*"

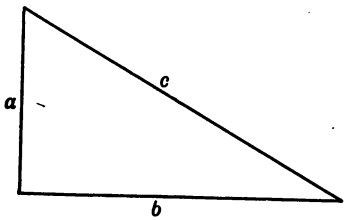


FIG. 41.

The legs of the right triangle are usually represented by "a" and "b" and the hypothenuse by "c."

Stated as a formula the above theorem is

$$a^2 + b^2 = c^2.$$

NOTE.—It is also proved in geometry that if the square of one side of a triangle is equal to the sum of the squares of its other two sides, the triangle is a right triangle, the right angle being opposite the greatest side.

Exercises

Find the missing parts of the following right triangles:

1. a = 3, b = 4, c = ?
2. a = 5, b = 12, c = ?
3. a = ?, b = 24, c = 25.
4. a = 18, b = 80, c = ?
5. a = 9, b = ?, c = 15.
6. a = 10, b = ?, c = 26.
7. a = ?, b = 15, c = 17.
8. a = ?, b = 40, c = 41.

9. What is the hypothenuse of a right angle whose legs are 510 ft. and 680 ft. respectively?

10. A house 25 ft. wide has a gable 10 ft. high. Find the length of a rafter.

11. Find the length of the diagonal of a rectangle 25 ft. × 15 ft.

12. Rectangular frames are often braced by a piece from one corner to the opposite lower corner of the frame. Find the length of a brace for such a frame 8 ft. × 6 ft.

13. A telegraph pole 30 ft. high is supported by a guy wire fastened 10 ft. from its top and anchored 25 ft. from its base. What is the length of the guy wire?

LESSON XLIII

REVIEW

1. Find the distance across corners of each of the following square head screws:

$\frac{1}{4}$ in.; $\frac{3}{8}$ in.; $\frac{3}{4}$ in.; $\frac{7}{8}$ in.; $\frac{5}{16}$ in.

2. Find the distance across corners of each of the following square bolt heads:

$1\frac{1}{4}$ in.; 2 in.; $4\frac{1}{4}$ in.; $2\frac{3}{4}$ in.; $3\frac{1}{2}$ in.

3. A piece of round stock 2 in. in diameter is to be milled square on one end. What is the side of the largest square that can be cut from the piece?

4. Thirty feet from the mast head of a derrick a boom is fastened to the mast. The boom is 25 ft. long. One foot from the mast head is a block containing two pulleys and $\frac{1}{2}$ ft. from the end of the boom is a block with a single pulley. How long a rope must be used to allow the boom to stand at right angles with the mast and also to have 10 ft. of rope below point of attachment of boom to mast?

5. The width of a rough, U. S. standard nut or bolt head in terms of the diameter (D) of the bolt is $W = 1\frac{1}{2}D + \frac{1}{8}$ in. Find the width of a square nut for a bolt whose diameter is $1\frac{1}{8}$ in., also the distance across corners.

6. If the base of a rectangle is three times its altitude and its perimeter is 40, find its base, its altitude, its diagonal and its area.

7. The area of a rectangle is 288 sq. ft. Find its base if its altitude is twice its base.

8. Solve for the letters:

$25 + 3X = 125$, $2Y + 10 + 5Y = 79$, $2X^2 + 125 = 375$.

$36 + X^2 = 136$, $5Y + 6Y - 10 = 84$, $40 + 3A^2 = 770$.

9. Find the length of the straight line from the home plate to second base of the baseball diamond.

10. Two persons M and P start from vertex A to go to the opposite vertex C of the square ABCD. M goes by way of B along the perimeter of the square and P on the diagonal AC. If they both reach C at the same time what is the ratio of their speeds, the side of the square being 100?

LESSON XLIV

FACTORING

1. The factors of a number are the numbers that multiplied together will produce the number; *e.g.*, the factors of 8 are 2, 2 and 2.

2. What are the factors of 16? Of 49? Of 18? Of 24? Of 100? Of 27?

3. Numbers that are produced by squaring a number are called perfect squares; *e.g.*, 25 the square of 5 is a perfect square.

4. Name all the perfect squares between 1 and 144.

5. Factor each of the following so that one of the factors is a perfect square:

75; 50; 125; 150; 18; 27; 45; 63; 8; 12; 20; 24; 98; 147; 128; 192; 32; 48; 80; 96; 288; 432.

6. Find all the factors of each of the following:

150; 275; 95; 100; 240; 3675; 5625; 2468; 357; 7564; 231; 440; 1111.

7. The product of three numbers is 720 and two of the numbers are 8 and 9; find the other number.

8. What three numbers multiplied together will give 144? 1728?

LESSON XLV

SQUARE ROOT

1. Does the $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$? Does the $\sqrt{25 \cdot 4} = \sqrt{25}\sqrt{4}$? Does the $\sqrt{25 \cdot 16} = \sqrt{25}\sqrt{16}$? Does the $\sqrt{36 \cdot 25} = \sqrt{36}\sqrt{25}$?

2. These problems illustrate the following principle:

The square root of the product of two numbers is equal to the product of their square roots.

That is $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

3. Find the square root of 27.

$$\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\,\sqrt{3} = 3\sqrt{3} = 3 \times 1.732 = 5.196$$

4. Find the square root of a^2b.

$$\sqrt{a^2 b} = \sqrt{a^2}\sqrt{b} = a\sqrt{b}.$$

5. Find the square root of each of the following:

18; 8; 12; 75; 50; 32; 48; 72; 108; 98; 147.

6. Simplify:

$\sqrt{c^2d}$; $\sqrt{c^2xy}$; $\sqrt{b^2d^2k}$; $\sqrt{p^4d}$; $\sqrt{a^2b^2c}$; $\sqrt{m^2n^2}$.

7. Simplify:

$\sqrt{20}$; $\sqrt{100}$; $\sqrt{200}$; $\sqrt{300}$; $\sqrt{80}$; $\sqrt{125}$; $\sqrt{1875}$; $\sqrt{150}$.

8. $5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = ?$ $8\sqrt{5} + 6\sqrt{5} - 3\sqrt{5} = ?$ $7\sqrt{3} + 8\sqrt{3} + 6\sqrt{3} = ?$ $15\sqrt{7} - 8\sqrt{7} - 7\sqrt{7} = ?$ $9\sqrt{6} + 3\sqrt{6} - 12\sqrt{6} = ?$ $18\sqrt{19} + 10\sqrt{19} - ? = 0$ $15\sqrt{3} + 10\sqrt{3} - ? = 0$ $8\sqrt{5} + 7\sqrt{5} - ? = 0$.

9. $\sqrt{50} + \sqrt{98} - \sqrt{32} = ?$ $\sqrt{80} - \sqrt{45}\,\sqrt{500} = ?$ $\sqrt{75} + \sqrt{27} + \sqrt{48} = ?$ $\sqrt{24} + \sqrt{150} + \sqrt{216} = ?$ $\sqrt{200} + \sqrt{800} + \sqrt{3200} = ?$ $\sqrt{432} + \sqrt{363} - \sqrt{300} = ?$ $\sqrt{3} + \sqrt{8} + \sqrt{27} = ?$ $\sqrt{5} + \sqrt{20} + \sqrt{12} = ?$

LESSON XLVI

REVIEW

1. Factor so that one of the factors shall be a perfect square:

500; 800; 288; 1440; 112; 250; 360; 490.

2. Solve for the value of the letter in each of the following:

$5x + 6x = 22$; $8x + 10 = 30$; $7x - 5x = 24$; $7x + 15 = 50$; $3x + 5x + 10 = 14$; $4x + 7x - 5x = 25$; $10x + 8x + 12 = 38$; $6x - 5x + 7x = 36$.

3. Simplify:

$\sqrt{1440}$; $\sqrt{250}$; $\sqrt{360}$; $\sqrt{490}$; $\sqrt{3a^2}$; $\sqrt{2a^2}$; $\sqrt{5b^2}$; $\sqrt{9c^2}$.

4. If the hypothenuse of a right triangle is 2a and one of the legs is a, what is the other side?

5. If each leg of a right triangle is m, what is its hypothenuse?

6. Each side of a square is t; find its diagonal.

7. If the diagonal of a square is $15\sqrt{2}$, what is its side?

8. If the area of a square is 144, find the diagonal.

9. What is the area of a circle whose radius is 12?

10. The area of a circle is 25π, what is its radius?

Solution.—$\pi R^2 = 25\pi$; $R^2 = 25$; $R = 5$.

11. Find the radius of each of the following circles whose areas are:

36π; 49π; 64π; 100π; 625π; 144π.

12. Find the diagonal of a square whose area is 100; 400; 900; 625; 169; 576.

13. What is the area of a square whose diagonal is $8\sqrt{2}$; $6\sqrt{2}$; $3\sqrt{2}$; $14\sqrt{2}$; $16\sqrt{2}$; $7\sqrt{2}$; $10\sqrt{2}$; $a\sqrt{2}$; $b\sqrt{2}$?

LESSON XLVII

TRIANGLES

1. A triangle with two sides equal is an isosceles triangle. The angles opposite the equal sides are called base angles of the isosceles triangle.

A triangle with all its sides equal is an equilateral triangle.

2. Construct from cardboard an isosceles triangle, tear off one of the base angles. Apply to the other base angle. Are they equal?

What statement can you make about the angles of an equilateral triangle?

3. If one base angle of an isosceles triangle contains 18°, how many degrees in the other base angle? If 23°? If 45°? If 60°? If 48°?

4. Construct from cardboard a triangle with no two sides equal. Cut off two of the angles and place them as in the following figure:

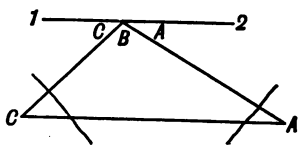


FIG. 42.

What kind of angle is now formed at B? How many degrees in it? What then is the sum of the angles of a triangle? The sum of the angles of a triangle is . . . . .

5. Find the third angle of each of the following triangles:
$A = 28°$, $B = 48°$, $C = ?$, $A = 30°$, $B = 60°$, $C = ?$, $A = 45°$, $B = ?$, $C = 90°$, $A = 40°$, $B = 60°$, $C = ?$, $A = 60°$, $B = 60°$, $C = ?$, $A = 100°$, $B = 40°$, $C = ?$

6. In a right triangle what is the sum of the two acute angles?

7. If one acute angle of a right triangle is 30°, what is the other acute angle? If 20°? If 48°? If 45°?

8. The vertex angle of an isosceles triangle is 100°. How many degrees in each base angle?

9. If a base angle of an isosceles triangle is 80°, how many degrees in the vertex angle?

10. In a triangle $\angle A = 2 \angle B$ and $\angle C = 3 \angle B$. How many degrees in each angle of the triangle?

HINT.—Let x = number of degrees in $\angle B$.

11. In a certain triangle $\angle A = 2\angle B$ and $\angle C = 2\angle A$. Find the number of degrees in each angle of the triangle.

12. Find the number of degrees in the sum of the angles of a quadrilateral.

13. Construct a triangle with a pair of angles equal. Test to find out if the sides opposite those angles are equal.

14. Construct an equilateral triangle. Draw its altitudes. Test to find out if the altitude of an equilateral triangle bisects the base to which it is drawn.

LESSON XLVIII

THE 30° RIGHT TRIANGLE

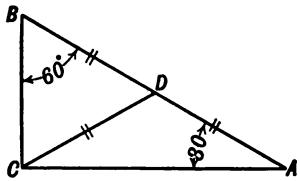


FIG. 43.

Theorem.—*In a 30° right triangle the leg opposite the angle of 30° is one-half the hypothenuse.*

Given the right triangle ABC with C the right angle, $\angle A = 30°$, BC the side opposite 30° and AB the hypothenuse.

To Prove.—$BC = \frac{1}{2}AB$.

Proof.—In the $\angle C$ take $\angle DCA = 30°$.

Then $\triangle DAC$ is isosceles (?) $\therefore AD = DC$.

Also each angle of $\triangle DCB$ is 60° (?) $\therefore BD = DC$.

$$\therefore AD = BD, \text{ or } BD = \tfrac{1}{2}AB.$$

But $BC = BD$ (?) $\therefore BC = \frac{1}{2}AB$

Q. E. D.

Exercises

1. If in the above figure AB = 100, 120, 200, 300, 400, 18, 6a, 7b, 4(c + d), 8(e + f + g), what is BC?

2. If in a 30° right triangle the side opposite 30° is $15\frac{1}{2}$, what is the length of the hypothenuse?

3. If a kite string 200 ft. long makes an angle of 30° with the ground, about how high is the kite?

LESSON XLIX

REVIEW

1. If the hypothenuse of a right triangle with one of its acute angles 30° is 100, what is the length of the side opposite the angle of 60°?

2. A string 200 ft. long attached to the top of a pole reaches the ground at a point P. The angle made by the string and a line joining P with the foot of the pole is 30°. How high is the pole? How far is P from the foot of the pole?

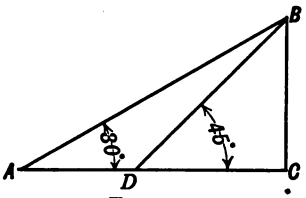


FIG. 44.

3. In this figure AB = 400 ft. Find BC, CD and DA. Also find number of degrees in ∠DBA and ∠BDA.

4. The diagonal of a rectangle is 30 ft. and makes an angle of 30° with the base. What is the altitude of the rectangle? What is the base? Its area?

5. A rope is stretched from an upper corner of a room 15 ft. square and 10 ft. high to the opposite lower corner. What is the length of the rope?

6. One rectangle is 40 ft. × 20 ft. and another rectangle is 80 ft. × 10 ft. Find the side of the square that has an area equivalent to the sum of the rectangles.

7. A circular cistern 8 ft. in diameter and 12 ft. deep is full of

water. If a pipe drains from it 10% of its contents in 1 hr. and another pipe conducts water to it equal to 8% of its contents in 1 hr., how many gallons of water will be in the cistern at the end of 1 hr.?

8. Your Street and Water Board placed 157 cylindrical street markers at the intersections of many principal streets. They are 9 ft. high and 4 in. in diameter. Find the cost of the paint to give them two coats, allowing 1 gal. to paint 250 sq. ft. one coat, the paint costing $2.50 per gal.

LESSON L

THE 60° RIGHT TRIANGLE

Theorem.—*In a* 60° *right triangle the side opposite* 60° *is one-half the hypothenuse multiplied by* $\sqrt{3}$.

Use Fig. 45 and show that $x = a\sqrt{3}$.

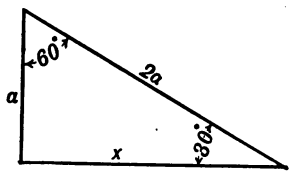


FIG. 45.

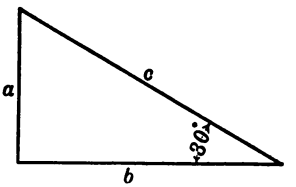


FIG. 46.

Exercises

1. If a = 10, c = ?, b = ? If $b = 8\sqrt{3}$, c = ?, a = ? If c = 20, a = ?, b = ? If a = 15, b = ?, c = ? If $b = 18\sqrt{3}$, c = ?, a = ? If c = 40, a = ?, b = ? (Fig. 46.)

2. The diagonal of a certain rectangle makes an angle of 60° with the base. If the base is $20\sqrt{3}$, what is the diagonal and also the area?

3. A ladder makes an angle of 60° with the ground and the foot of the ladder is $15\sqrt{3}$ ft. from the building against which it is leaning. How long is the ladder and how high does it reach on the wall?

4. If the width of a house makes an angle of 60° with the rafters and the rafters are 18 ft. long, how high is the gable and how wide is the house?

LESSON LI

REVIEW

1. Fig. 47 represents a lathe center with its dimensions. Find H and the diameter of the small end if the body tapers .6 in. per 1 ft.

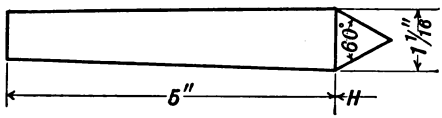


FIG. 47.

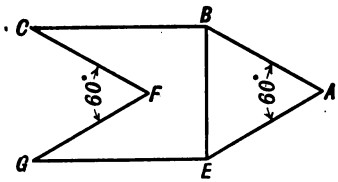


FIG. 48.

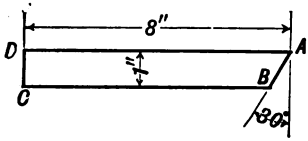


FIG. 49.

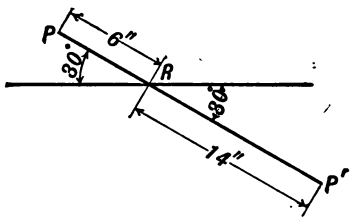


FIG. 50.

2. In Fig. 48 AB = AE = CF = FG = 1 in. If GE = 1.3 in., find the length of the figure, also its area.

3. Find length of AB also area of Fig. 49.

4. When the rocking lever of Fig. 50 is turned 30° from the horizontal line how much farther has P′ fallen than P has risen?

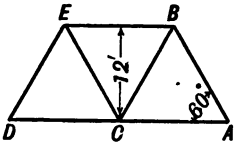


FIG. 51.

5. The diagram shows The Pratt truss used in bridge construction DE and BA are called struts. EB topchord. If the height of the truss is 12 ft.; find length of the struts, topchord and length of the bridge.

6. An inclined plane 20 ft. long makes an angle of 30° with the horizontal line. What is the height of the inclined plane?

7. If in the mortar box of Lesson 9, the inside length of the bottom is 6 ft. and the inside height of the sides is 14 in., find the inside top length of the box.

LESSON LII

ALTITUDE AND AREA EQUILATERAL TRIANGLE

1. Find the altitude of the equilateral triangle whose side is 8, 6, 10, 12, 14, 16, 18, 20.

2. The side of an equilateral triangle is 17. What is its altitude?

3. If the side of an equilateral triangle is a, show that its altitude is $\frac{a}{2}\sqrt{3}$. *Learn this formula.*

4. Use the result obtained in problem 3 and give the altitude if the side is 3, 4, 5, 6, 7, .87, 13.34, 8, 89.34, 25, 13.

5. Find the area of each of the triangles in problem 1 and 2.

6. Show that the area of the equilateral triangle whose side is a, is $\frac{a^2}{4}\sqrt{3}$. *Learn this formula.*

7. Use the result obtained in problem 6 and give the area of each of the following equilateral triangles:

5; 7; 8; 11; 13; 15; 2.5; 3.5; 4.68; 12; 16; 17; 22.2; 38; 40; 50.

8. The base of a prism is an equilateral triangle whose side is 5.4. Find the volume of the prism if its altitude is 6.

9. Find the volume of a prism whose altitude is h and whose base is an equilateral triangle side a.

LESSON LIII

THE REGULAR HEXAGON

If six equilateral triangles be arranged as shown in Fig. 52 a *regular hexagon* is formed.

The point (o) that is the common vertex of the equilateral triangles is the *center* of the hexagon. OB or OC, etc., is a radius of the hexagon. ∠ A, ∠ B, etc., are vertex angles of the hexagon.

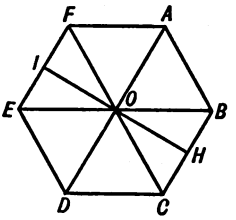


FIG. 52.

Exercises

1. How many degrees in each vertex angle of a regular hexagon.

2. Are the sides of a regular hexagon equal? Why?

3. Use problems 1 and 2 to form a definition of a regular hexagon?

4. What is the perimeter of a regular hexagon whose radius is 8, 9, 10, a, b, x?

5. How many degrees in ∠ BOE? Is BOE a straight line? Why? In ∠ IOH if OH is perpendicular to CB and OI is perpendicular to EF, is IOH a straight line? Why?

In screws that have hexagonal heads and also in hexagonal nuts the length of IOH is called the distance *across the flats*, the length of BOE the distance *across the corners*.

6. Find the distance across the flats of each of the following standard hexagonal nuts having given distance across the corners:

$\frac{1}{2}$ in.; $1\frac{3}{16}$ in.; $5\frac{1}{8}$ in.; $3\frac{7}{64}$ in.; $2\frac{29}{64}$ in.; $\frac{7}{8}$ in.; $2\frac{3}{32}$ in.

NOTE.—Use $\sqrt{3} = 1.7$ and express results as multiples of $\frac{1}{64}$ in., for example, the answer for $\frac{1}{2}$ in. is $\frac{6.8}{16}$ which is considered $\frac{7}{16}$ in.

7. Find the distance across the corners of each of the following standard hexagon nuts having given distance across the flats:

$\frac{1}{2}$ in.; $\frac{7}{8}$ in.; $1\frac{3}{8}$ in.; $3\frac{13}{16}$ in.; $2\frac{1}{2}$ in.; $1\frac{3}{4}$ in.

8. The distance across the flats for both a square and a hexagonal bolthead is 2 in.; find the distance across the corners for each and express the results correct to three decimal places.

9. Find the area of a regular hexagon whose side is 8, 10, 12, 13.5, $5\frac{1}{4}$; a, b, f, m.

10. A regular hexagon whose radius is 10 is inscribed within a circle; find area of part of the circle between the perimeter of the hexagon and the circumference.

11. Find the volume of a prism whose base is a regular hexagon, each side 20. The altitude of the prism is 8.5.

12. A piece of steel 2 in. in diameter and 10 in. long is milled into the largest possible hexagonal piece. Find its weight both before and after being milled.

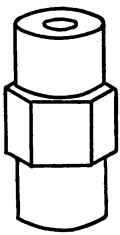


FIG. 53.

13. The distance across the flats of the sleeve is $3\frac{3}{8}$ in. and is $2\frac{1}{4}$ times the diameter of the hole. The hole is one-half the diameter of the cylinder and one-fourth the total length of the sleeve. The hexagonal part is one-third the total length. Find all dimensions and its weight when made of brass. 1 cu. ft. brass weighs 524.1 lb.

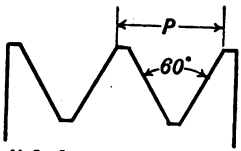
LESSON LIV

SCREW THREADS

1. The depth of a thread is the perpendicular distance from the bottom of the groove to the straight line joining the tops of the thread. Twice this distance is the double depth.

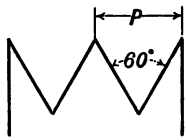
2. The root diameter of the screw is the outside diameter of the screw minus its double depth.

3. In practice there are threads of several shapes. We shall consider but two kinds, the Sharp V thread whose angle is 60° and the U. S. standard form of thread.



U.S. Standard Thread

FIG. 54.



60° V Thread

FIG. 55.

The U. S. standard has the same angle as the 60° thread, but has its top and its bottom flat. The width of this flat is one-eighth of the pitch of the thread. The depth of this thread is three-fourths of the depth of the 60° V thread of the same pitch.

EXERCISES

(These problems refer to Screws of Single Thread)

1. A screw has 15 threads per in. What is its pitch? Its lead.

2. Find the pitch and lead for the following number of threads per inch:

13; 14; 25; 36; 8; 17; 18; 20.

3. Determine the depth of each of the following 60° V threads whose pitches are $\frac{1}{12}$ in., $\frac{1}{16}$ in., $\frac{1}{10}$ in., $\frac{1}{8}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., $\frac{1}{5}$ in.

4. Show that the depth of the 60° V thread is equal to its pitch multiplied by $\frac{1}{2}\sqrt{3}$ or about .866.

5. Find the root diamters of each of the screws in problem 3 if the outside diameter of each is 2 in.

6. Determine the depth of thread for the pitches given in problem 3 for the U. S. standard-shaped threads.

7. How many revolutions will a 60° V thread make in advancing 2 in., if its pitch is $\frac{1}{10}$ in.?

8. If the flat of a U. S. standard-shaped thread is $\frac{1}{8}$ in., what is its depth?

9. If the root diameter of a 60° V thread with 10 threads per in. is $1\frac{1}{4}$ in., what is its outside diameter?

10. Find the outside diameter of the U. S. standard with 8 threads per in. and a root diameter of $1\frac{3}{4}$ in.

LESSON LV

REVIEW

1. How many revolutions will a 60° V thread make in advancing $1\frac{1}{4}$ in. if its pitch is $\frac{1}{20}$ in.?

2. What is the depth of the thread in problem 1?

3. If the flat of a U. S. standard-shaped thread is $\frac{3}{16}$ in., what is its depth?

4. The root diameter of a 60° V thread with 12 threads to the inch is $\frac{3}{4}$ in. What is its outside diameter?

5. The outside diameter of a U. S. standard is 1 in. and its pitch $\frac{1}{4}$ in. If the screw is 2 in. long what is the length of the thread?

6. If the outside diameter of a sharp V thread is $\frac{1}{2}$ in. and its pitch $\frac{1}{10}$ in. what is the length of the thread of a screw 3 in. long?

7. If the feed is $\frac{1}{32}$ in. and the cutting speed 20 ft. per min., find the required time for the tool to advance 2 in. along a piece 3 in. in diameter.

8. When cutting hard steel the speed may be 33 ft. per min. if the depth of the cut is $\frac{1}{4}$ in. and the feed $\frac{1}{16}$ in. Find the number of R.P.M. a cylinder of hard steel 2 in. in diameter is making for the above data.

9. If the scale is $\frac{1}{8}$ in. = 5 ft. 0 in. what is the length on a working drawing for each of the following:

2 ft. 6 in.; 4 ft.; 10 ft.; 12 ft. 8 in.; 25 ft.; 48 ft.?

If the scale is $\frac{1}{4}$ in. = 3 ft. 0 in.? 1 in. = 5 ft. 6 in.? $\frac{3}{4}$ in. = 4 ft. 6 in.?

10. A carpenter has a mitre box 5 in. wide outside dimensions. Explain how he will find the points on the box through which he must saw to get a 45° angle; a 60° angle.

11.

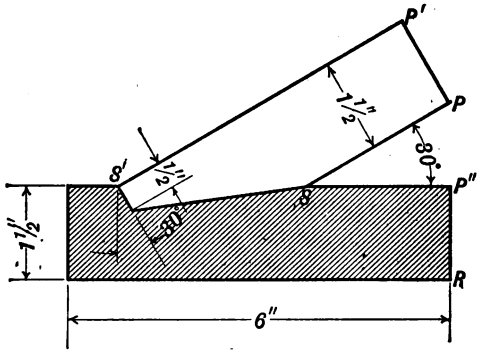


FIG. 56.

The above figure is a side view of an oblique thrust. R, P″ and P are in a straight line. Find area of shaded portion, length of SP and S′P′.

12. Find the nearest number of 64ths to which each of the following fractions is equivalent:

$$\frac{15}{17};\ \frac{11}{13};\ \frac{6}{7};\ \frac{9}{11};\ \frac{21}{23};\ \frac{125}{127};\ \frac{11}{15};\ \frac{19}{21};\ \frac{13}{14}.$$

13. State the limit of the error for each of your answers in problem 12.

14. If $R = 5$ and $r = 3$, find value of V in

$$V = \frac{4}{3}\pi\,(R^3 - r^3).$$

15. If $r_1 = 6$, $r_2 = 3$, and $h = 2.4$ find value of V in

$$V = \pi\frac{h}{2}(r_1^2 + r_2^2) + \frac{\pi h^3}{6}.$$

16. Use the following formula

$$T = \frac{[(S \times P) + 1386]D}{86{,}670}$$

to find the value of T in the table

T	D	S	P	T	D	S	P
	7	5	100		18	5	180
	7	5	200		10	5	120
	8	5	160		16	5	140
	12	5	2220		9	5	200

17. How many degrees in each angle of the triangle ABC if $\angle A = 3\angle B$ and $\angle C = 2\angle A$?

18. Simply:

$$\sqrt{75} + \sqrt{108} - \sqrt{27};\ \sqrt{50} + \sqrt{200};$$
$$\sqrt{300} - \sqrt{200} + \sqrt{12};\ \sqrt{32} + \sqrt{48}.$$

19. If the feed is $\frac{1}{16}$ in. and the work has a speed of 30 R.P.M.. how long will be required to cut a piece 1 foot long?

20. Floor moulding is required for a floor $36' \times 24'$. If the strips can be purchased $10'$, $12'$ or $16'$ long, which length would you purchase assuming no waste and the price the same per foot? Why?

21. What is the side of an equilateral triangle whose area is $25\sqrt{3}$? $20\sqrt{3}$? $100\sqrt{3}$?

22. Solve for the letter in each of the following:

$5y^2 + 10 = 110$; $\quad 25 + a^2 = 75$; $\quad 15 + b^2 = 115$.
$5p^2 + 7 = 232$; $\quad 36 + k^2 = 436$; $\quad 29 + d^2 = 173$.
$3t^2 + 6 = 426$; $\quad 10 + e^2 = 210$; $\quad 17 + g^2 = 417$.

LESSON LVI

THREAD CUTTING

Mathematically, *thread cutting* is merely a problem of gears.

The cutting tool is made to advance along the piece being cut by means of a lead screw. The piece is made to revolve as the tool advances. If the number of R.P.M. of the piece is the same as the R.P.M. of the lead screw, the tool will evidently cut on the piece the same number of threads per inch as the number on the lead screw; *e.g.*, if the lead screw has 4 threads per in. the tool will cut 4 threads per in. on the piece. If the R.P.M. of the piece is twice the R.P.M. of the lead screw, then 8 threads per in. will be cut.

The diagram (Fig. 57) of the head stock of a simple geared lathe will help to make clear how the number of R.P.M. of the work and of the lead screw is accomplished.

I, I′ and I″ are intermediates, hence have no effect on R.P.M. I and I′ determine the direction of rotation of the lead screw. C and S′ are both keyed to the stud shaft. S′ and S in the problems that follow are equal. C and L are the gears that by changing determine the R.P.M. of the lead screw.

Standard "change gears" are made in series that have a common difference or either 4 teeth or 5 teeth. The smallest gear of each series contains 20 teeth, and the largest 120 teeth and 100 teeth respectively.

20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, . . 120, . . A series.
20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, . . 100, . . B series.

Lathes are also supplied with other gears as 46, 66, 69.

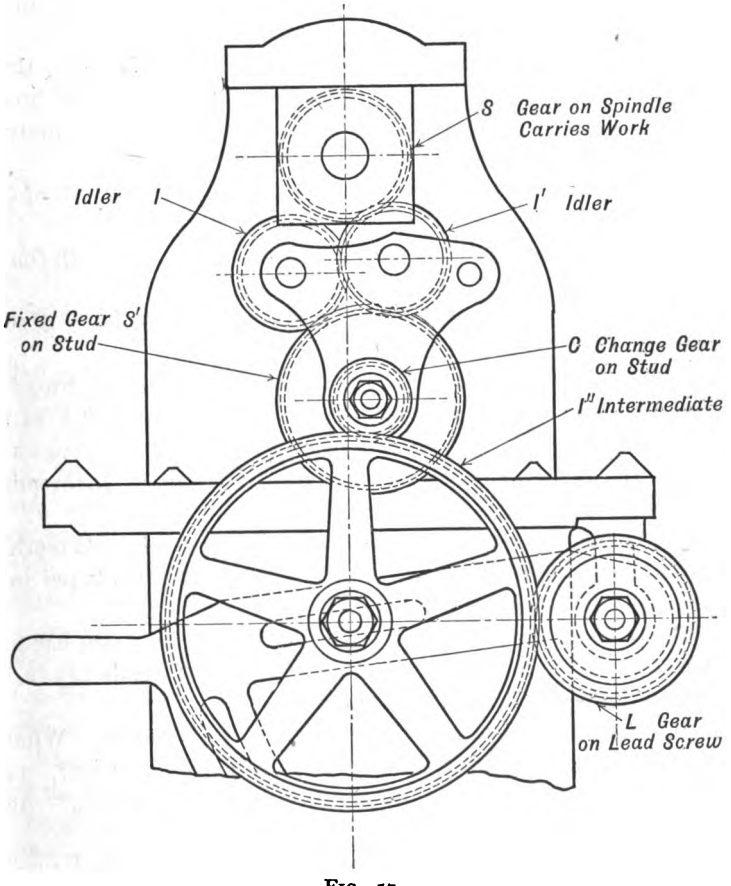

S Gear on Spindle Carries Work
Idler I
I' Idler
Fixed Gear S' on Stud
C Change Gear on Stud
I'' Intermediate
L Gear on Lead Screw

FIG. 57.

Exercises

1. In the above diagram S is made to revolve; explain how the motion is communicated to L.

2. S = S′. If the number of teeth of C equals twice the number of teeth of L which is revolving the faster and how many times as fast? If L has 6 threads per in., how many would be cut on the work?

3. If C = L and S = ½S′, compare number of R.P.M. of C with that of L.

4. If C = ⅓L and S = ½S′ compare R.P.M. of C with that of L.

5. If C = L, S = S′ and I″ = 3L, compare R.P.M. of C with that of L.

6. If the lead screw has 8 threads per in. how many threads per inch are being cut on a piece making twice as many R.P.M.? 3 times as many? 2½ times as many? How many times as many to cut 20 threads per in.? 30 threads per in.? 40 threads per in.? 12 threads per in.?

7. The lead screw has 4 threads per in., and L has 80 teeth. How many teeth must C have in order that 8 threads per in. may be cut? 16 threads per in.?

8. What gears on C and L from the A series may you use to cut 12 threads per in. with a lead screw of 6 threads per in.? 5 threads per in.? 8 threads per in.?

9. The lead screw of a lathe has 5 threads per in. What gears on C and L may you use to cut 10 threads per in.? 12 threads per in.? 15 threads per in.? 3 threads per in.? 18 threads per in.?

10. With a lead screw of 5 threads per in., what is the largest number of threads per inch that can be cut in a simple-geared lathe with S = S′?

11. The standard pipe thread for ¾-in. pipe is 11½ threads per in. What gear will you use to cut this thread with a lead

screw of 6 threads per in.? 5 threads per in.? 4 threads per in.?

12. Let N_T = No. of teeth of L, N_R = R.P.M. of L, N'_T = No. of teeth of S, N'_R = R.P.M. of S. Then show that

$$\frac{N_T}{N'_T} = \left(\frac{N'_R}{N_R}\right) = \frac{\text{Threads per inch to be cut}}{\text{Threads of L}}.$$

13. Use the formula of problem 12 to find number of threads that will be cut per inch when $N'_T = 30$, $N_T = 20$ and $L = 6$; $N'_T = 40$, $N_T = 45$, $L = 4$; $N_T = 65$, $N'_T = 55$, $L = 8$; $N_T = 120$, $N'_T = 48$; $L = 5$; $N_T = 100$, $N'_T = 85$, $L = 8$; $L = 6$, $N_T = 60$, $N'_T = 30$; $N'_T = 85$, $N_T = 80$, $L = 8$; $L = 6$, $N_T = 52$, $N'_T = 40$.

Very often the number of teeth of S and of S′ are not equal, the number of S being less than the number of S′. The effect of this is to make S revolve more rapidly than S′ and therefore than L provided C and L are equal. Suppose S = 20 and S′ = 40; then S will make two revolutions while S′ and L are making one. Hence, if L has 6 threads per in., 12 threads will be cut on the work. That is, we get the same result with S = 20, S′ = 40 and L = 6, as if we had S = S′ and L = 12. This illustrates the following: *When S and S′ are not equal, we assume C and L to be equal, find the number of threads the given lead screw will cut per inch, use this number as the lead screw and proceed as in the previous work.* This number is called the *lead number* or the *calculating lead screw.*

Example.—If S = 30; S′ = 40, and L = 6, what gears may be used to cut 12 threads per in.?

Solution.—Since S/S′ = ¾, S will make 1⅓ revolutions, while L is making one, that is, 6 × 1⅓ or 8 threads per in. will be cut. Hence the lead number is 8. Also $1\frac{2}{8} = \frac{48}{32}$; therefore gears of 48 and 32 must be used, 32 on C and 48 on L.

Stated as a formula the above is

$$\frac{\text{Threads per inch to be cut}}{\text{Lead number}} = \frac{N_T \text{ of L}}{N_T \text{ of C}}.$$

Many lathes are made with the number of teeth of S and of S′, 30 and 40 respectively.

Exercises

1. If S = 30, S′ = 40, find the lead number when L = 4, L = 6, L = 8, L = 5.

2. If S = 30, S′ = 40 and L = 6, what gears may be used to cut 10 threads per in.? 15 threads per in.? 16 threads per in.? 20 threads per in.? 4 threads per in.? 40 threads per in.?

3. If S = 30, S′ = 40 and L = 6, what gears may be used to cut a standard pipe thread for ¾-in. pipe?

4. With S = 30, S′ = 40, L = 6 and the A series, what is the largest number of threads per inch that can be cut in a simple-geared lathe? The least number?

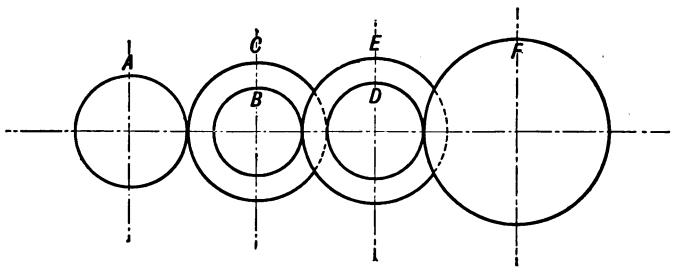


FIG. 58.

In the above train of gears B and C are keyed to the same shaft, also D and E. Such a train is said to be compounded. If the train is set in motion by A or A drives C, then A is called the driving gear of the train and F the driven gear. Also A, B, D are called the drivers and C, E, F the driven.

Law for Compound Gears.—

N_A = R.P.M. of A and N_F = R.P.M. of F

A, B, C, etc. = the number of teeth of each gear.

Then

$$\frac{N_A}{N_F} = \frac{C.E.F}{A.B.D}.$$

Expressed in words this formula is: *The R.P.M. of the driving gear divided by the R.P.M. of the driven gear of a train is equal to the product of the number of teeth of the driven gears divided by the product of the number of teeth of the driving gears.*

Exercises

1. If $N_A = 10$, $A = 20$, $B = 18$, $C = 30$, $D = 22$, $E = 26$ and $F = 40$. Find N_F.

2. Solve problem 1 without the use of the formula.

3. Find N_A when $A = 10$, $C = 20$, $B = 15$, $E = 30$, $D = 18$, $F = 36$ and $N_F = 20$.

4. Find N_F when $N_A = 8$, $A = 12$, $C = 24$, $B = 18$, $E = 36$, $D = 20$ and $F = 40$.

Compound-geared Lathe.—A lathe is said to be compound geared when there are two changes of speed between S′, the fixed gear of the studshaft, and L, the gear of the lead screw. Fig. 59 shows one way of compounding the gears of a lathe.

C′ is another gear keyed to the same shaft with I″. The change gears are L, C′, I″ and C. I″ and C′ are together known as the compound and one gear is usually twice the other, as 60 and 30. Compound gearing is used when in a simple-geared lathe a gear with a larger number of teeth than is usually made would be needed or a gear too large for the center distance between the shafts.

Law for Compound-geared Lathe.—

$$\frac{\text{Threads to be cut per inch}}{\text{Lead number}} = \frac{I.''L}{C'.C} = \frac{I''}{C'.} \times \frac{L}{C}.$$

The following will illustrate how to apply this formula.

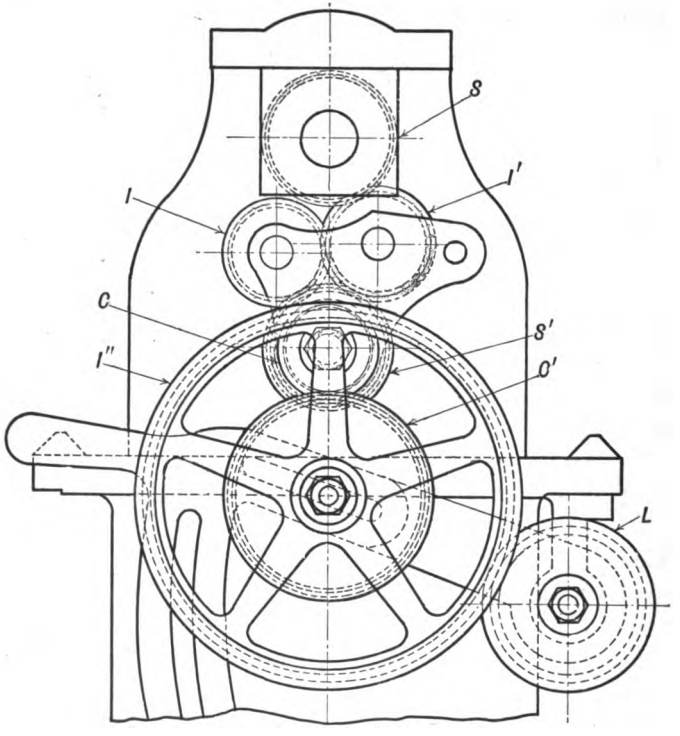


FIG. 59.

Find the gears necessary to cut $11\frac{1}{2}$ threads per in. with a lead screw of 6 threads per in. on a lathe with S = 30 and S′ = 40.

Lead number = 8; then $\frac{11\frac{1}{2}}{8} = {}^{23}\!/_{16} = {}^{23}\!/_{32} \times {}^{2}\!/_{1} = {}^{46}\!/_{64} \times {}^{60}\!/_{30}$. That is $\frac{I''}{C'} = \frac{60}{30}$ and $\frac{L}{C} = \frac{46}{64}$, the required gears.

Since I″ and C′ are usually 60 and 30, we should try to make

the fraction equal to the product of two fractions, one of which is either $\frac{1}{2}$ or $\frac{2}{1}$.

Thread Cutting

1. With S = S′; find the compound gearing necessary to cut 32 threads per in., with a lead screw of $\frac{1}{5}$-in. pitch.

2. With S = 30 and S′ = 40, find the compound gearing that may be used to cut 22 threads per in. with a lead screw of pitch $\frac{1}{6}$ in.

3. With S = 30, S′ = 40 and L = $\frac{1}{5}$-in. pitch, what simple gearing will cut 23 threads per in.? What compound gearing?

4. Make L = 8 threads per in. in problem 3 and then solve it.

5. Make L = 6 threads per in. in problem 3 and cut 27 threads per in.

6. With S = S′ and L = $\frac{1}{5}$-in. pitch, find gears for cutting 24 threads per in. by compounding.

7. Make S = 30 and S′ = 40 in problem 6 and solve it.

LESSON LVII

TAPER

A piece of turned work with uniformly increasing diameter is called a taper.

The difference of the diameters of the two ends of the taper is the taper of the piece. It is usually given as a certain number of inches per foot or per inch. For example, if a taper is 1 ft. long with diameters of 2 in. and 1 in. respectively, the taper is 1 in. per ft. or $\frac{1}{12}$ in. per in.

Exercises

1. If the larger diameter of a taper 18 in. long is 3 in. and the taper $\frac{1}{8}$ in. per in., what is the smaller diameter?

2. The diameters of a taper are 2 in. and 1 in. If the taper is $\frac{1}{16}$ in. per in., find the length of the taper.

3. What is the difference in the diameters of a taper 10 in. long whose taper is $\frac{7}{8}$ in. per ft.?

4. What is the taper per foot of a taper, $3\frac{1}{2}$ in. with diameters $\frac{7}{8}$ in. and $\frac{7}{10}$ in.?

5. The taper per foot of a Jarno taper is .6 in. Find the length of a Jarno taper whose diameters are $\frac{7}{8}$ in. and $\frac{7}{10}$ in.; 2 in. and 1.6 in.; 1 in. and .8 in.

6. The length of a Jarno taper is 10 in. and its larger diameter 2.5 in.; find its smaller diameter. If the length is $9\frac{1}{2}$ in. and the smaller diameter 1.9 in., what is the larger diameter?

7. The taper per foot of a Brown and Sharp taper is $\frac{1}{2}$ in. Find the length if the two diameters are 2.25 in. and 2.58 in.

If the length is $7\frac{3}{4}$ in. and the larger diameter 2.052 in., find the smaller diameter.

8. The diameters of a taper are 1.045 in. and 1.348 in. and the taper per foot .516 in.; find the length of the taper.

9. What is the taper per foot of a piece $9\frac{1}{16}$ in. long with diameters of $1\frac{7}{8}$ in. and $2\frac{1}{8}$ in.?

10. If the length of a taper is L ft., and the diameters of its ends a in. and b in., what is its taper (T) per foot?

11. Use the formula to find T when L = 2, A = 1, B = 2; when L = 6 in., A = 1, and B = 144; when L = 10 in., A = 2, B = 3.375; when T = .623, A = 1.02 and B = 1.28. Find L.

LESSON LVIII

TAPER TURNING

Tapers are turned in a lathe either by means of the taper attachment or by offsetting the tailstock. When the taper attachment is used the taper in inches per foot is determined and the taper attachment set to that number. If the tailstock is offset we must know the taper per foot and the length of the piece expressed in feet.

Then if S = offset in inches, T = taper in inches per foot and L = length of piece in feet, we have as the formula for the offset, or

$$S = \frac{TL}{2}$$

NOTE.—The proof of this formula depends upon principles of geometry.

Exercises

1. A cylinder 1 ft. long is to be tapered $\frac{1}{4}$ in. per ft. How much must the tailstock be offset?

2. Determine the offset for each of the following:

Taper $\frac{3}{8}$ in. per ft., piece 8 in. long; taper $\frac{1}{2}$ in. per ft., piece 18 in. long; taper .6 in. per ft., piece 10 in. long; taper .602 in. per ft., piece 14 in. long; taper .625 in. per ft., piece 5 in. long; taper .592 in. per ft., piece 25 in. long.

3. The tailstock of a lathe is offset 2 in. Find the taper if the piece is 8 in. long; 10 in. long; 2 ft. long.

4. If the offset is $\frac{3}{8}$ in. and the taper $\frac{1}{2}$ in. per ft., what is the length of the piece?

5.

How much must the tailstock be offset to turn the piece shown in the figure?

FIG. 60.

6. A lathe center 5 in. long is to be tapered .6 in. per ft. Find the offset.

7. A taper pin 4½ in. long has the large end .49 in. in diameter and the small end .398 in. in diameter. How much was the tailstock offset to turn the piece?

8.

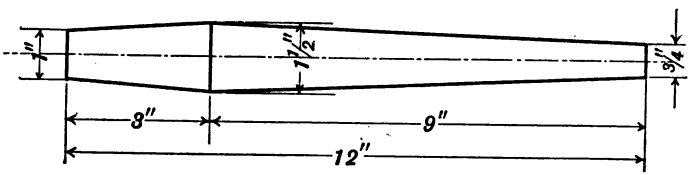


FIG. 61.

Determine each offset for turning the piece shown.

9. Seller's taper is ¾ in. per ft. Determine the length of a piece being turned to this taper if the set over is ⅝ in.

10. The Jarno taper No. 18 is 9 in. long and has diameters of 1.8 in. and 2¼ in. What must be the offset to turn this taper?

11. How far must the tailstock be set over to taper a piece 15 in. long with American taper? (A. T. = 9/16 in. per ft.)

LESSON LIX

REVIEW

1. Find the altitude of the equilateral triangle whose side is ½ in., ¾ in., ⅜ in., correct to three decimal places.

2. A boiler tube is to be made 3 in. in inside diameter, 1/16 in. thick and 10 ft. long. How long a piece of brass 3 in. square will be required to make the tube?

3. Find the length of the line ABCD as in the figure.

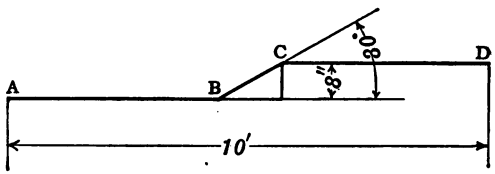


FIG. 62.

4. Find the altitude of each of the equilateral triangles of Fig. 63.

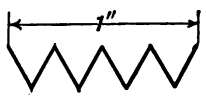


FIG. 63.

5. In turning a locomotive wheel 78 in. in diameter, what is the proper number of revolutions per minute, in order that the cutting speed may be 10 ft. per min.?

6. A piece of brass 4 in. in diameter is making 80 R.P.M. What is the speed of a point on its surface?

7. If a piece tapers .0026 in. per in., what is its taper per foot?

8. The standard pipe thread taper is ¾ in. per ft. How much must the tailstock be offset to turn this taper on a piece 2 ft. long?

9. A round shaft is 3¼ in. in diameter. Find the length of the greatest square end that can be made on the shaft.

10. If S = 30 and S′ = 40 and L = ⅛-in. pitch, find the simple gearing you may use to cut 5 threads per in.; also the compound gearing.

11. If the scale is 1″ = 0′ 8″ what should be the length of each line for a scale drawing of problem 3.

12. What is the micrometer reading for each of the following: .777 in.? .326 in.? .565 in.? .480 in.? .444 in.? .345 in.?

13. What is the nearest number of 64ths for each number of problem 12?

14. The diameter of a drill is 2 in. Its speed is 92 R.P.M. and its feed per revolution is .015 in. How many cubic inches are being removed per minute?

15. A drill 4 in. in diameter making 46 R.P.M. removes 10.8 cu. in. per min. Find its feed per revolution.

LESSON LX

RATIO

The quotient obtained by dividing a by b is called the ratio of a to b. The ratio of a to b is written as a:b, or a/b or a÷b.

Exercises

1. Find the value of each of the following ratios:

10:5; 16:20; 32:8; 100÷25; 17÷19; 25÷5; $^{17}/_{34}$; $^{34}/_{17}$; $^{1728}/_{144}$; a^2/a; b^4/b^2; c^3/c^2; d^6/d^3.

2. Find the ratio of the areas of two rectangles of altitudes 5 and 10 and bases 8 and 16 respectively.

3. What is the ratio of the areas of two triangles of altitudes 10 and 18 and bases 22 and 30 respectively?

4. The radii of two circles are 10 ft. and 15 ft. What is the ratio of their areas? Of their circumferences?

5. Two gears of 60 and 45 teeth respectively. What is the ratio of their speeds? If they have a and b teeth?

6. Find the ratio of the speeds of two pulleys connected by a belt if their diameters are 12 in. and 8 in.; 15 in. and 7 in.; a and b.

7. What is the ratio of the areas of two circles whose radii are p and q?

8. What is the ratio of the surface speeds of the pulleys in problem 6?

9. The ratio of the speeds of the driving gear to the driven gear is 3:2. If the driving gear containd 48 teeth, how many teeth has the driven gear?

10. The *efficiency* of a machine is the ratio of the units of work given out by the machine and utilized to the total number of units of work put into it. What is the efficiency of a machine that gives out 37 ft.-lb. from 50 ft.-lb. put into it? 20 ft.-lb. from 45 ft.-lb.? 38 ft.-lb. from 48 ft.-lb.?

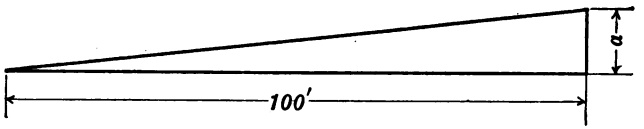


FIG. 64.

11. The ratio a/100 is called the grade of the slope. The 100 ft. is horizontal distance and a the vertical.

If a road bed rises 10 ft. each 100 ft. measured horizontally, what is its grade? What is a 7% grade? 10% grade?

12. A vertical rise of 20 ft. in 1000 ft. is what grade?

13. Water consists of 2 parts hydrogen and 1 part oxygen. What is the ratio of oxygen to hydrogen? Of hydrogen to oxygen? Of hydrogen to water? Oxygen to water?

14. Name several gears that will have a speed ratio of 5:2.

15. The volumes of two spheres have the same ratio as the cubes of their radii. Find the ratio of the volumes of two spheres whose radii are 2 and 3; 4 and 5; and 7 and 9; 10 and 12.

16. Find the ratio of the areas of two squares whose sides are m and n respectively.

17. What is the ratio of the lead to the pitch of a single-thread screw?

18. If the scale is $\frac{1}{4}$ in. = 2 ft. 0 in., what is the ratio of a scale drawing 2 in. long to the length of the line it represents?

19. A speed ratio of 7:3 is required for two gears. If the driven gear has 56 teeth how many must the driving gear have?

20. If pigiron contains 93% pure iron, 3% carbon and 2% sulphur, find the ratios of the different elements given.

21. The heating surface of a certain boiler is 1800 sq. ft. The grate measures 9 ft. $\times$ 8 ft. Find the ratio of grate surface to heating surface.

22. If the ratio of two numbers is 9: 7 and one of the numbers is 1423, what is the other number? *Two solutions.*

LESSON LXI

SECTORS AND SEGMENTS

The part of a circle between two radii and an arc is called a sector of the circle, as AOB.

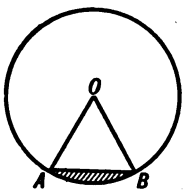


Fig. 65.

The part of a circle between an arc and a chord is called a segment of the circle, the shaded part of Fig. 65.

The angle at the center of the circle formed by the two radii is called the angle of the sector.

Exercises

1. What part of the whole circle is a sector whose angle is 60°? 120°? 30°? 90°? 180°? 45°? 15°? 38°?

2. Find the area of a sector of 90°, if the radius of the circle is 10, 8, 12, 5, 25, 15, 35.

3. If the radius of a circle is 10, find the area of a sector of 60°, 90°, 45°, 30°, 180°, 36°, 18°.

4. If the area of a sector AOB is 28 and the triangle AOB is 20, what is the area of the segment?

5. Find the area of the segment if the area of sector AOB is 98.3 and triangle AOB 76.84.

6. State how we can find area of a segment of a circle.

7. Find the area of each of the segments in the following, having given that the angle AOB is 60°:

Radius of circle 8; 9; 10; 12; 15; 22; 30; 42.

LESSON LXII

REVIEW

1. The radius of each circle is 5. Find the area included between the circles of Fig. 66.

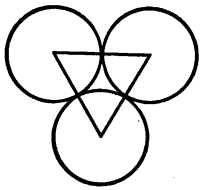


FIG. 66.

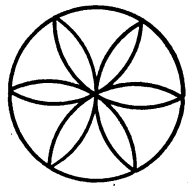


FIG. 67.

2. The diameter of a circle is 10. The circumference is divided into six equal parts and lobes formed as in figure. Find the area of each lobe of Fig. 67.

3. The diameter AE of Fig. 68 is divided into four equal parts, and semicircles drawn as indicated. Find the area of each figure. Let AE equal 8.

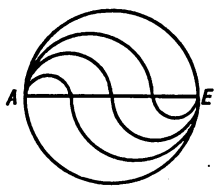


FIG. 68.

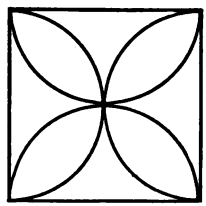


FIG. 69.

4. Find the area of each lobe of Fig. 69 if the side of the square is 16.

LESSON LXIII

REVIEW

1. One cubic inch of steel weighs .29 lb. An I-beam has a cross-section as shown in Fig. 70 and a length of 12 in. Find its weight.

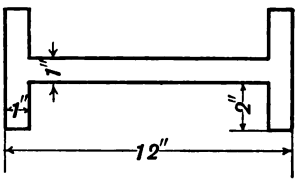


FIG. 70.

2. In forging a bolt 24 in. in total length $1\frac{1}{4}$ in. in diameter with a head $\frac{1}{2}$ in. thick and $1\frac{1}{2}$ in. square, the stock is cut from a bar of iron, $1\frac{1}{2}$ in. square in cross-section. How long a piece will it take? Allow 1 in. in length for waste.

3. A circular disk $\frac{1}{2}$ in. thick and 4 in. in diameter is to be made from the same rod. Find the length required.

4. The external diameter of a hollow cast-iron shaft is 18 in. and its internal diameter is 10 in. Calculate its weight if the length is 20 ft. and cast iron weighs .26 lb. per cu. in.

5. Find the length of steel wire in a coil, if its diameter is .025 in., and its weight 50 lb.

6. The larger diameter of a piece of steel is $\frac{3}{4}$ in., and the smaller $\frac{35}{64}$ in. Find the taper per foot if the piece is $1\frac{5}{8}$ ft. long.

7. The larger diameter of a piece of steel is $\frac{1}{2}$ in. If it is $1\frac{3}{4}$ ft. long and the taper is $\frac{3}{4}$ in. per ft., what is the smaller diameter? Find the offset for turning this taper; also the taper of problem 6.

8. Find the cost of 25 pieces of 2 in. × 8 in. each 16 ft. long at $30 per M.

9. It is required to build a bin that will hold 50 bu. It can be built in a space 7 ft. long and 3 ft. 6 in. wide. How high must it be if a bushel contains $1\frac{1}{4}$ cu. ft.?

10. A certain lumber company has a piece of timber 3 ft. square and 80 ft. long. How many board feet in the piece?

LESSON LXIV

REVIEW

1. Find the area between the tangents to the circle and the arc. The tangents make an angle of 60°, the radii are perpendicular to the tangents and OF bisects angle APB. OA = 12 in.

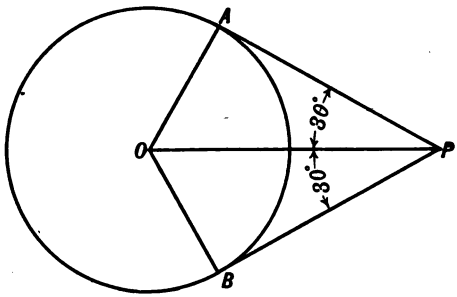


FIG. 71.

2. Find the area included between the four circles. Each radius is 5. ABCD is a square.

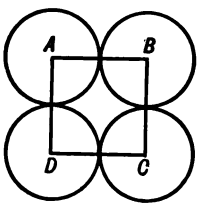


FIG. 72.

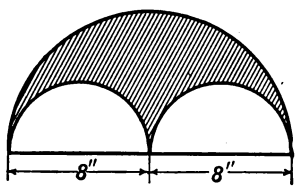


FIG. 73.

3. The diameter of the circle is 16. Semicircles are drawn as indicated in Fig. 73. Find area of shaded portion.

4. The center of one circle lies on the circumference of the

other. If the radius of each circle is 12, find the area common to both circles. Fig. 74.

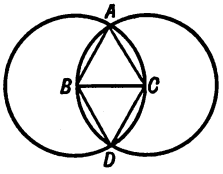


FIG. 74.

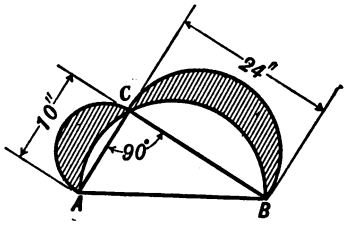


FIG. 75.

5. Find the area of the two crescents formed as given in Fig. 75. ACB is a right triangle.

LESSON LXV

AREA OF THE SURFACE OF A PYRAMID AND OF A CONE

The line from the vertex of a pyramid perpendicular to its base is called the altitude of the pyramid. The foot of this altitude in such pyramids as we shall study is the middle of the base. Name the altitude of the pyramid (Fig. 76).

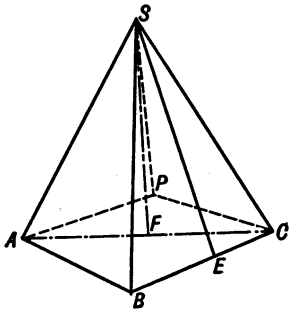


FIG. 76.

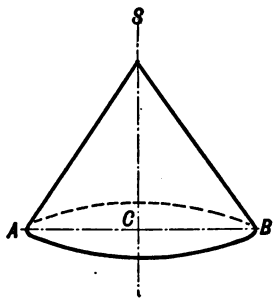


FIG. 77.

The altitude of any one of the triangles that form the faces of the pyramid is called the slant height of the pyramid. What is the slant height of the above pyramid?

What is the altitude of the cone? The slant height?

The area of the curved surface of a cone is one-half the product of its slant height by the circumference of its base? How do you find the area of the surface of a pyramid?

Find the area of the lateral surface of each of the following pyramids:

1. Base a square 20 in. each side, slant height 18 in.

2. Base a square 18 in. each side, slant height 20 in.

3. Base a square 4 ft. 5 in. each side, slant height 3 ft. 8 in.

4. Base an equilateral triangle each side a, slant height b.

5. Find the complete area of each of the above pyramids.

6. If each side of the base of a square pyramid is 8 in. and its altitude 6 in., what is its slant height? Its area?

7. Find the area of the curved surface of each of the following cones:

Radius of base 12 in., slant height 16 in.; radius of base 10 in., slant height 20 in.; radius of base 3 ft., slant height 10 ft.; radius of base 2 ft., slant height 4 ft.; radius of base 5 ft. 4 in., slant height 12 ft. 6 in.; radius of base 4 ft. 5 in., slant height 8 ft. 9 in.

8. The altitude of a cone is 8 in. and the radius of its base 6 in. What is its slant height? Its area?

9. The slant height of a cone is 1 ft. 8 in., and the radius of its base 6 in. What is the area of its surface? Its complete area? Its altitude?

10. Find the complete area of a cone whose slant height is 24 in., the radius of its base being 8 in.

11. The radius of the base of a cone is 5 in., and its slant height makes an angle of 60° with the radius. Find the complete area of the cone.

LESSON LXVI

VOLUME OF A PYRAMID AND OF A CONE

A pyramid is one-third of a prism with the same base and altitude as the prism.

A cone is one-third of a cylinder with the same base and altitude as the cylinder.

How then will you find the volume of a pyramid? Of a cone?

1. Find the volume of each of the following pyramids:

Base a square 10 ft. on each side, altitude 15 ft.; base a square $12\frac{1}{2}$ ft. each side, altitude 24 ft.; base a square 9 ft. 8 in. each side, altitude 10 ft.; base a square 7 ft. 3 in. each side, altitude 7 ft., base an equilateral triangle each side 10 in. and altitude 20 in.

2. What is the volume of a pyramid whose altitude is 18 ft. 6 in. and whose base is a right triangle, hypothenuse 10 ft. and one acute angle 30°?

3. What is the weight of a solid steel pyramid 18 in. long and the base a square 10 in. on each side?

4. Find the volume of each of the following cones:

Radius of base 8 in., altitude 12 in.; radius of base 12 in., altitude 15 in.; radius of base 10 in., altitude 3 ft.; radius of base 3 ft. 3 in., altitude 1 ft. 6 in.; radius of base 3 ft., altitude 3 ft.; radius of base 4 in., altitude 4 ft. 2 in.; radius of base r, altitude h.

5. Find the volume in cubic inches of the following round piece:

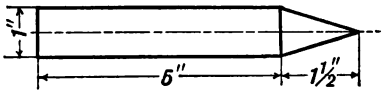


FIG. 78.

6. A cylindrical piece of steel 1 ft. 4 in. long and 4 in. in diameter has a conical hole 4 in. long and 3 in. in diameter bored from one end of it. What is the weight of the piece?

7. Find the volume of a cone 10 in. in diameter if it tapers ½ in. per in.

8. Find the volume of a cone 1 ft. 6 in. long if it tapers 1 in. per in.

LESSON LXVII

REVIEW

1. A pile of coal of conical shape 10 ft. high lies at an angle of 30° with the horizontal. How many tons in it if 1 cu. ft. weighs 38 lb.?

2. Find the weight of a conical casting of iron 8 in. in diameter and slant height 14 in.

3. The rain which falls on a house 22 ft. × 36 ft. is conducted to a cylindrical cistern 8 ft. in diameter. How great a fall of rain would it take to fill the cistern to a depth of $7\frac{1}{2}$ ft.?

4. How many gallons of water will a 6-in. pipe deliver per hour if the flow is 3 ft. per sec.?

5. A band saw runs on pulleys 48 in. in diameter at a rate of 180 R.P.M. If the pulleys are decreased 18 in. in diameter, how many R.P.M. will they have to make to keep the band saw travelling at the original speed?

6. A shaft has upon it two pulleys, each 8 in. in diameter. The speed of the shaft is 400 R.P.M. What must be the size of the pulleys of two machines if, when belted to these shaft pulleys, one of them has a speed of 300 R.P.M. and the other 900?

7. A $\frac{3}{4}$-in. drill, cutting cast iron, may cut at the rate of 40 ft. per min. How many R.P.M. may it make?

8. Find the cost at 40¢. per lb. for sheet copper to line bottom and sides of a cubical vessel 7 ft. each edge, if the sheet copper weighs 12 oz. per sq. ft.

9. If the feed is $\frac{1}{16}$ in. and the work has a speed of 164 ft. per min., how long will it take to cut a piece 2 in. in diameter and 1 ft. long?

10. If S = S′ and L = ½-in. pitch, find the simple gearing that may be used to cut 15½ threads per in.

11. If the threads of problem 10 is a 60° V thread, find its depth.

12. The ratio of the areas of two circles is 1:4 and the radius of the smaller circle is 6. What is the radius of the larger circle?

13. Studding for partitions is 2 in. × 4 in. and 16 ft. long. It is set 16 in. between centers. How many pieces must be bought for a partition 8 ft. high and 12 ft. long? What will it cost at $30 per M?

14. A right cone of altitude 10 ft. has a slant height of 18 ft. Find its complete area and also its volume.

LESSON LXVIII

REVIEW

1. A main line shaft runs 176 R.P.M., a pulley on this shaft is 36 in. in diameter and is belted to a pulley on the countershaft 12 in. in diameter. Another pulley on this same countershaft is 16 in. in diameter and is belted to a pulley 4 in. in diameter on a grinder. What is the speed of the countershaft? Of grinder spindle? If the grinding wheel is 10 in. in diameter, what is its surface speed?

2. The diameter of a driving pulley is 9 in. and its speed is 1000 R.P.M. What is speed of driven pulley whose diameter is 4 in.? If this speed is too fast, what should be the diameter of the diven pulley to have a speed 250 R.P.M. less than the 4-in. pulley? If the speed of the 4-in. pulley is too slow by 250 R.P.M., what size driving pulley should be used instead of the 9-in. pulley? If we keep both pulleys (9 in. and 4 in.) and make our speed changes by changing speed of 9-in. pulley, what would be the speed of 9-in. pulley to give 1125 R.P.M. of 4-in. pulley?

3. How many gallons of water in a railway track tank 1200 ft. long, 19 in. wide and 7 in. deep, if the water is 2 in. below the top of the tank?

4. The diameters of the steps of a step cone pulley are 8 in., $5\frac{1}{2}$ in. and 4 in. respectively. Find the ratio of their surface speeds when the shaft to which the pulley is attached is making 900 R.P.M.

5. A pump has a water cylinder of 6 in. and a stroke of 16 in. How many gallons of water are pumped in 1 hr. if the pump makes 60 strokes per min.?

6. A smokestack 90 ft. high is to be held in place by five guy wires attached 30 ft. from the top of the stack. The wires are anchored 55 ft. from the bottom of the stack on a level with its bottom. Find the number of feet in the guy wires allowing 35 ft. for fastening.

7. One formula for making concrete is 1 part cement, 2 parts sand and 4 parts crushed stone. How many cubic feet of each will be required to make a concrete wall 100 ft. long, 2 ft., thick and 4 ft. high.

8. What is the micrometer reading for a piece of iron whose diameter is .422? About how many 64ths of an inch?

9. Each edge of a pyramid of four faces is 8 in. Find its complete area.

10. Find the value of the letter in each of the following:

$3x + 8x - 6x + 4x = 25$; $\frac{1}{2}x + \frac{1}{3}x + \frac{2}{5}x = 10$;

$7x + 2x + 3x - 8x = 15$; $\frac{1}{5}x + \frac{1}{4}x + \frac{1}{10}x = 30$;

$2x + 3x + 8x - 2x = 14$; $\frac{1}{4}x + \frac{3}{4}x + x = 28$.

LESSON LXIX

REVIEW

1. A steel plate 5 ft. long, 3 ft. 6 in. wide and 1 in. thick, has a hole 10 in. in diameter cut through it. Find weight of plate, allowing .29 lb. per cu. in.

2. A tubular boiler has 124 tubes each $3\frac{7}{8}$ in. in diameter and 18 ft. long. What is the total tube surface?

3. A room is heated by steam pipes. There are 240 ft. of 2-in. pipes and 52 ft. of 5-in. pipes and 2 ft. of $4\frac{1}{4}$-in. feed pipe. What is the total heating surface of the room?

4. A hollow steel shaft 10 ft. long is 18 in. in external diameter and 8 in. in internal diameter. Find weight of the shaft.

5. If the depth of a sharp V thread angle 60° is $\frac{1}{2}$ in., what is its pitch?

6. Three circles each of 9-in. radius are tangent to each other. Find area between the three circles.

7. Find the volume generated by an equilateral triangle whose side is 8 if it revolves about its altitude as an axis. Find area also.

8. If the back stay of a suspension bridge is 125 ft. long and is anchored 120 ft. from the base of the pier, what is the height of the pier?

9. The distance across the flats of a 2-in. Whitworth hexagon nut is $3\frac{5}{32}$ in. Find the distance across the corners.

10. A pipe has a sectional area of 125 sq. in. at 1 part and 80 sq. in. at another. If 6000 cu. ft. of water flow past each section per hour, find the velocity of the water in feet per second, at each point.

11. A triangular piece of steel is $\frac{3}{4}$ in. thick and two of its sides are 28 in. and 32 in. If these sides form an angle of 30°, what is the weight of the piece?

12. Solve each of the following:

$6a^2 + 10 = 160$; $2b^2 + 3b^2 = 500$; $8b^2 - 100 = 700$;
$10n^2 - 10 = 90$; $8k^2 - 15 = 225$; $6x^2 + 4 = 40$.

LESSON LXX

REVIEW

1. Find the weight of 100 steel planer bolts with heads 1 in. square and $\frac{3}{8}$ in. thick, body 5 in. long and diameter $\frac{5}{8}$ in.

2.

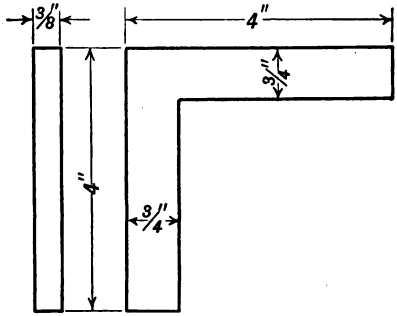


FIG. 79.

Calculate the amount of stock in the angle weld (Fig. 79).

3.

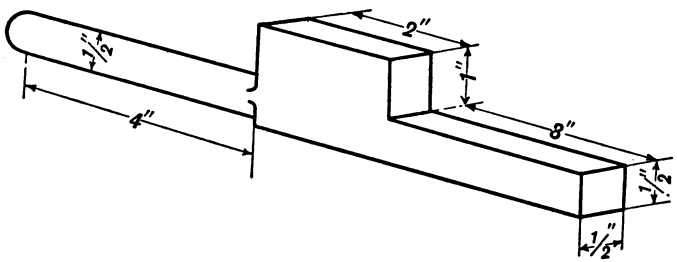


FIG. 80.

Calculate the length of stock $\frac{1}{2}$ in. $\times$ 1 in. required for the forging shown in the diagram.

4.

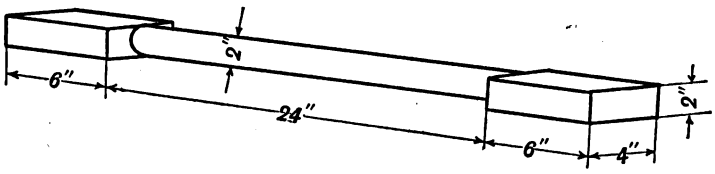


FIG. 81.

Find the weight of the forging and the length of stock 2 in. × 4 in. required to make it. 1 cu. in. weighs .2779 lb.

5.

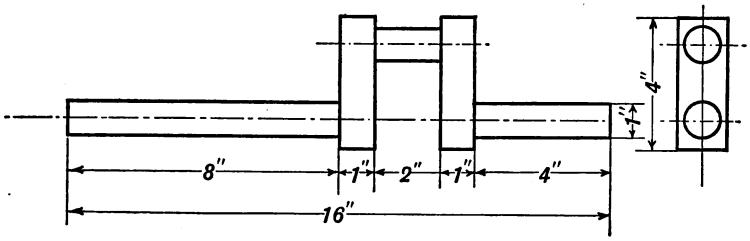


FIG. 82.

Find the weight of the forged crankshaft as per diagram.

6.

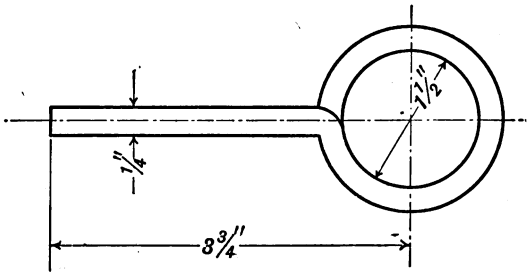


FIG. 83.

Calculate the amount of stock in the eye bend.

7.

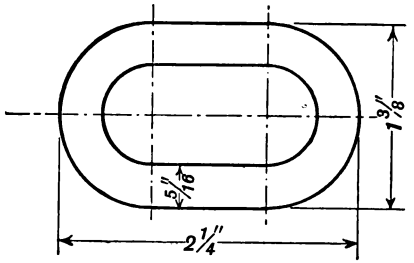


FIG. 84.

The link is forged from $\frac{5}{16}$-in. round stock. Find the weight of a chain of 50 steel links.

LESSON LXXI

REVIEW

1. An engine pumping water from a cylindrical tank 10 ft. 6 in. in diameter lowered the surface of the water 2 ft. 4 in. What was the weight of the water pumped out?

2. What is the weight of a piece of steel shafting 12 ft. long and 3 in. in diameter?

3. A brass plate 2 in. thick, in the shape of a semicircle with a radius of 14.5 in., has four holes through it, each $\frac{3}{4}$ in. in diameter. What is its weight? 1 cu. in. of brass weighs .3031 lb.

4. A rectangular box is 8 ft. 8 in. long, 5 ft. 2 in. wide and 4 ft. 3 in. high. Find the cost at 12¢. per lb. of lining the sides and bottom with lead weighing 7 lb. per sq. ft.

5. Two pulleys each 18 in. in diameter are connected with a belt. How long is the belt if the distance between their centers is 10 ft.?

6. The water cylinder of a pump is $6\frac{1}{4}$ in. in diameter and the length of the stroke is $15\frac{1}{2}$ in. Find the time such a pump would require to empty a rectangular cistern 30 ft. 6 in. long, 13 ft. 9 in. wide and 12 ft. 4 in. deep, if the pump is making 90 strokes per min.?

7. The angle of elevation of the top of a flag staff is 30° at a point P. If the distance from P to the foot of the staff is 60 ft., how high is the staff?

8. When a house is heated with a hot air furnace the cross-sectional area of the cold air box should equal three-fourths the total cross-sectional area of the hot air pipes supplying the rooms? What should be the area of the cold air box for eight hot air pipes, two being 10 in. in diameter and the others each 8 in. in diameter?

LESSON LXXII

REVIEW

1. A poker handle is made from ¼-in. stock. The ring is 1 in. in diameter inside measurement. How much stock must be allowed for the making of ring? The length is calculated from center of stock.

2. The wheels of a band saw are 36 in. in diameter. What is the speed in feet per minute if the wheels are making 500 R.P.M.; if 300 R.P.M.?

3. If the small diameter of a taper shank is ¾ in. and the taper is .6 in. per ft., what would be the large diameter of the shank if it were 4 in. long?

4. If a forge uses 30 lb. of coal per day and there are 23 forges in the shop, what is the cost of fuel at $4.50 per ton to run the shop 200 school days?

5. How many feet per minute does a point on the surface of a 2½-in. cylinder travel, if the cylinder is making 1200 R.P.M.? 600? 3000?

Find the cost of the following order for lumber used in making imposing tables.

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
16	Chestnut	3½″× 3½″×37″		7¢.	
8	Chestnut	⅞″×10⅛″×34″		7¢.	
8	Chestnut	⅞″×10⅛″×22″		7¢.	
8	Chestnut	⅞″×3⅛″ ×34″		7¢.	
8	Chestnut	⅞″×3⅛″ ×22″		7¢.	
4	Chestnut	⅞″×24″ ×36″		7¢.	
8	Chestnut	⅞″×6⅛″ ×24″		7¢.	
8	Poplar	⅞″×6⅛″ ×24″		6¢.	
4	Poplar	⅜″×6⅛″ ×24″		6¢.	
4	Poplar	⅜″×24″ ×24″		6¢.	

Find the total number of board feet and the cost of the following mill bill for doors for a case in the stock room.

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
14	Chestnut	1″×2½″×66½″		7¢.	
8	Chestnut	1″×2½″×50″		7¢.	
8	Chestnut	1″×2½″×37″		7¢.	
22	Chestnut	1″×2½″×14½″		7¢.	
15	Chestnut	1″×3″ ×14½″		7¢.	

Find the total number of board feet in the following mill bill for indian clubs:

No. of pieces	Wood	Dimensions	Bd. ft.	Price	Cost
200	Chestnut	2″ ×4⅛″×19″		7¢.	
400	Chestnut	1¼″×3¾″×8″		7¢.	

LESSON LXXIII

ALLOYS

An alloy is a mixture obtained by fusing metals with each other. When two or more metals are fused we obtain a new metal that often has properties exhibited by none of the metals in the combination nor by any single metal.

For example, gold, which in its pure state is too soft and flexible for use as coins, jewelry, etc., is hardened by mixing with copper in the ratio of 9 parts gold to 1 part of copper. Brass is an alloy of copper and zinc, harder than copper and easier to work.

Unless a small quantity of lead is added to brass it cannot be used in turning operations, since the tool will tear and not cut it. An alloy of 50% bismuth, 30% lead and 20% tin has the property of melting at a much lower temperature than any of the metals in the combination.

In industrial work metals that have certain peculiar properties, are needed for special purposes. Type metal must have the property of making sharp distinct lines; likewise pattern metal. This is accomplished by adding antimony to the alloy. Antimony when used as a part of the alloy causes the metal to expand on cooling; hence to fill out the corners of the molds making a pattern with sharp lines.

Antimony also renders the alloy harder. The bearings for machinery must be antifrictional, that is, not easily heated by contact with the revolving machinery. They must be hard and strong particularly for heavy work; for high-speed machinery softer bearings may be used.

A fairly good bearing for high-speed machinery contains

80% lead and 20% tin, but is not very hard. A metal widely used for heavy machinery bearings contains 80.5% lead, 11.5% tin, 7.5% antimony and .5% copper. If a metal is required that melts at a low temperature bismuth is used as part of the alloy. The plugs of automatic sprinklers and doors used for fire protection are made of such an alloy.

Alloys containing sodium have the peculiar property of producing by oxidation, material which will saponify with the oil used in the bearing and thus assist lubrication. Thus by combining copper, lead, zinc, etc., in different ratios we often produce a new metal that answers the peculiar needs of various industrial problems.

Exercises

1. In the foundry the formula for "yellow brass castings" contains 66⅔% copper and 33⅓% zinc. How many pounds of each must be used to make 1500 lb. of yellow brass?

2. How many pounds of copper are used with 200 lb. of zinc to make yellow brass?

3. Recently in your foundry 44 lb. of copper, 3 lb. zinc, 2 lb. tin and 1 lb. of lead were used to make a bronze casting. Determine the per cent. of each element used.

4. How many pounds of each element must be used to make 1800 lb. of bronze according to the formula in problem 3?

5. How many pounds of bronze will 100 lb. of tin make?

6. In making pattern metal we use 50% tin, 45% zinc and 5% antimony. How much pattern metal will 90 lb. of zinc make? 28 lb. of antimony? 500 lb. of tin?

7. A pattern that weighs 10 lb. will contain how much of each of the elements of pattern metal?

8. Fire sand is about 98% silica. What is the weight of the silica in 5000 lb. of fire sand?

9. A pile of fire sand contains 1 ton of silica. How many tons of sand in the pile?

10. A certain grade of molder's sand is 76% silica and 4% aluminum. How much of the sand will contain 758 lb. of silica? How much aluminum in the sand?

11. Another grade of molder's sand is 86% silica and 8% aluminum. How much silica in a pile of the sand that contains 872 lb. of aluminum?

12. One of the bronze castings made in your foundry weighed 28.5 lb. How many pounds of tin in it? Of lead?

13. A certain alloy contains 50 lb. of copper and 25 lb. of zinc. Write the formula for this alloy in per cents.

LESSON LXXIV

ALLOYS

1. How many pounds of carbon in a ton of cast iron that contains 2.75% carbon?

2. If tool steel is 1.25% carbon how many pounds of tool steel will contain 25 lb. of carbon?

3. Naval brass is 62% copper, 1% tin and 37% zinc. Find the amount of each in 1200 lb. of naval brass. U. S. Navy Department.

4. Hard bronze for piston rings is 22% tin and 78% copper. How many pounds of hard bronze will contain 2340 lb. of carbon? U. S. Navy Department.

5. An alloy of 88% tin, 8% antimony, 3.5% copper and .5% bismuth is used for the bearings of high-speed dynamos. Calculate the amount of each in 2450 lb. of such alloy.

6. The bearings for railway trucks contain 42% tin, 56% zinc and 2% copper. How much tin and copper must be used to make such an alloy that contains 672 lb. of zinc?

7. Babbitt metal for high pressure bearings is 90% tin, 7% antimony and 3% copper. Find the amount of antimony and copper in such a composition that contains 1800 lb. of tin.

8. One ton of babbitt metal adapted for low pressure and medium speed contains 160 lb. tin, 400 lb. antimony and 1440 lb. lead. Find the per cent. of each for this composition.

9. One thousand pounds of plastic metal contain 800 lb. tin, 100 lb. lead, 10 lb. antimony, 80 lb. copper, and 10 lb. bismuth. Find the per cent. of each in this alloy.

10. Find the amount of each element in 2500 lb. of the alloys in problems 8 and 9.

11. The U. S. Navy Department uses brazing metal that

is 85% copper and 15% zinc. How many pounds of each will be required to make 3200 lb. of this alloy?

12. At $13 per 100 lb. for copper and $5 for zinc, find the cost of the alloy in problem 11.

13. Use the following to find the cost of as many different alloys given above as possible: Cost per 100 lb.—lead $4; zinc $5; antimony $9; copper $13; and tin $30.

LESSON LXXV

THE PRINT SHOP

The units of length in the print shop are the *inch*, the *pica*, the *nonpareil* and the *point*.

The dimensions of cards, sheets of paper, etc., are expressed in inches. The pica, which is $\frac{1}{6}$ in. is used to measure the lengths of printed matter, *e.g.*, the dimensions of a piece of printed matter is usually expressed as 13 picas wide and 20 picas long, or 20 picas wide by 30 picas long, etc. Sometimes, however, the width is expressed in picas and the length in inches, *e.g.*, a newspaper column 13 picas wide and 20 inches long.

The *nonpareil* is one-half of a pica, and the *point* is one-twelfth of a pica.

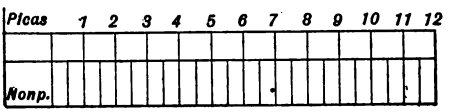


FIG. 85.

The body of metal type is measured by the point, as 16-point type, 36-point type, etc. A 24-point type means the height of the type is 24 points or $24\frac{1}{72}$ in. Metal type is made in the following number of points:

Common sizes: 6, 8, 10, 12, 14, 18, 24, 30, 36, 48, 60, 72, 84, 96, 120.

Odd sizes, chiefly book and newspaper sizes: $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 7, 9, 11.

Sizes rarely used: 16, 20, 22.

6-POINT 10-POINT 12-POINT

FIG. 86.

The amount of type in any composition is measured by a square whose side is any number of points. This unit is called the EM. The number of ems in any body of printed matter corresponds to the area of a rectangle. When measuring printed matter set in 8-point type the side of the em is 8 points, set in 10-point type the side of the em is 10 points, etc.

Exercises

1. How many picas in 3 in.? $4\frac{1}{2}$ in.? $\frac{3}{8}$ in.? 6 in.? 24 in.? How many nonpareils? How many points?

2. In the following number of points find the number of picas, inches, and nonpareils:

360; 24; 320; 144; 100; 168.

3. Find the number of square points in an 8-point em; a 10-point em, a 6-point em; an a-point em; a y-point em.

4. What is the value of each of the following ratios:

8-point em: 6-point em; 10-point em: 16-point em; 12-point em: 36-point em?

5. How many 8-point ems in each of the following ems:

640 10-point ems? 3200 12-point ems? 1280 16-point ems?

6. At the same rate per M which will cost the most, a page set in 8-point type or 10-point type or 16-point type? Why?

7. An edition of a certain newspaper had seven columns 13 picas wide and 21 in. long on each page. How many 12-point ems per page? How many 10-point ems?

8. The body of the printed matter on the page of a certain book is 20 picas wide and 33 picas long. How many 4-point ems per page? Find the cost of setting the page at 50¢. per M 8-point ems.

9. A double newspaper column is $26\frac{1}{2}$ picas wide and 24 in. long. Find the cost of setting $2\frac{1}{2}$ such columns at 54¢. per M 8-point ems.

10. How many ems, and what size on the page of this book? Answer the same questions about your text-book in English.

11. If 130 ems of composition contain 50 words, how many ems in an article of 2000 words? 2500 words? 1600 words?

12. How many lines set solid of 18-point type in 1 in.? In 4 in.? In 12 in.? In $5\frac{1}{2}$ in.? In $10\frac{3}{4}$ in.? In a in.? In b in.? In an ordinary newspaper column?

13. How many lines per inch set solid of the sizes of type in common use?

14. What size type is used to set solid 9 lines per in.? 7 lines per in.? 12 lines per in.? 2 lines per in.?

15. Find the width of a printed sheet of seven columns $12\frac{1}{2}$ picas wide with a margin of $\frac{3}{4}$ in. on each side if the columns are spaced with 6-point rule.

16. Measure several of your text-books to determine the size of the type used.

LESSON LXXVI

PRINT SHOP

1. A sheet of cardboard 22½ in. × 28½ in. is to be cut into tickets 2½ in. × 4 in. Find the greatest number of tickets that can be cut from the piece and tell how you will cut it.

2. How many sheets of cardboard 22½ in. × 28½ in. will be required to cut 1500 tickets 2½ in. × 4 in.? If the stock costs $2.50 per 100 sheets, how much will the tickets cost?

3. Your schedule cards are 4 in. × 5⅞ in. and were cut from cardboard 22½ in. × 28½ in. How many sheets were required to make 5000 of these cards? Find their cost at $2.40 per 100 sheets.

4. How many cards 2⅝ in. × 4⅜ in. can be cut from 50 sheets of the cardboard used in problem 3?

5. A card 2 in. × 7 in. was cut from cardboard 22 in. × 28 in. If the cardboard costs $2.40 per 100 sheets, find the cost of the cards per 1000.

6. Answer each of the above questions allowing 10% for press waste for each.

7. Letter paper sheets 16 in. × 21 in. are cut into letter heads 8 in. × 10½ in. How many sheets will be required for 2M such letter heads allowing 10% for press waste? What will the paper cost at $2.25 per ream of 500 sheets?

8. Which cuts to the better advantage sheets 17 in. × 22 in. or sheets 16 in. × 21 in., if letter heads 8 in. × 10½ in. are wanted? If letter heads 8½ in. × 11½ in. are wanted?

9. Use the size sheets that will be more economical for cutting half-size letter heads 8½ in. × 5½ in. and find the number of sheets you must use to print 5M such letter heads allowing 10% for press waste.

10. An order for 20M slips $3\frac{1}{2}$ in. $\times$ 2 in. was sent to your print shop. How shall these be set up in order that they may be printed with 2M impressions of the press? What will be the dimensions of the sheet on which they are printed? How many sheets will be required allowing 10% for press waste?

11. How many sheets of cardboard 22 in. $\times$ 28 in. will be required for backs used in padding the slips of problem 10, if 50 slips are put in each pad?

12. The cardboard back of a certain calendar is $8\frac{3}{4}$ in. $\times$ $6\frac{1}{4}$ in. If they were cut from sheets 22 in. $\times$ 28 in., how many sheets were required for 10,000 calendars allowing 10% for waste in printing?

13. How many cards 11 in. $\times$ 14 in. can be cut from 1000 sheets 22 in. $\times$ 28 in. If the sheets cost \$3 per 100, find the cost of the cards per 1000, adding 10% for waste in printing.

LESSON LXXVII

PRINT SHOP

1. Find the number of lines of 16-point type spaced with 2-point leads that can be set in 10 in.; in 15 in.; in 12 in.; in $5\frac{1}{2}$ in.; in 24 in.

2. The page of a certain book contains 33 lines. If the type is set solid and the composition is 33 picas long what is the size of the type used?

3. It is required to set solid 72 lines in a 12-in. space. What size type must be used? What size type must be used if the lines are spaced with 2-point leads?

4. Find the cost of setting the page in problem 2 at 55¢. per M ems.

5. If 65 ems of composition contain 25 words, how many ems in an article of 4000 words?

6. If 10,000 cards 4 in. × 6 in. are to be printed with 2500 impressions of the press, assuming no waste, how shall they be set up? What size sheets must be used? How long will it take to print them if the press averages 25 impressions per min.?

7. How long will it take to print the cards $19\frac{1}{2}$ in. × 14 in. that can be cut from 2000 sheets 22 in. × 28 in. if the press averages 1200 impressions per hr.?

8. The page of a certain book averages 34 words for each 84 ems. How many words in three pages of this book if the page is 21 picas × 33 picas?

9. Advertising announcements $8\frac{1}{2}$ in. × 12 in. are cut from sheets $22\frac{1}{2}$ in. × $28\frac{1}{2}$ in. Find the number of sheets required to print 150 of the "ads" if 10 are allowed for press waste.

10. Find the time required to print the letter heads 8 in. ×

10½ in. that can be cut from 800 sheets 16 in. × 21 in. when the press averages 18 impressions per min.

11. A certain job can be completed in 1 hr. 45 min. if the press averages 1200 impressions per hr. What must be the average number of impressions per hour to complete the job in 2 hr.? In 1 hr. 30 min.?

12. A piece of printed matter set solid 16 picas × 32 picas contains 1152 ems. Find the number of points for each em.

13. A certain page 20 picas × 30 picas contains 864 ems. What size type is used for the page if the work is set solid?

LESSON LXXVIII

PRINT SHOP

1. Common type is 60% lead, 30% antimony and 10% tin. How much of each in 400 lb. of type? 620 lb.? 157 lb.? 235 lb.?

2. The best type is 50% lead, 25% tin and 25% antimony. How many pounds of lead and of antimony must be melted with 75 lb. of tin to make this grade of type?

3. In a 200-lb. font of best type, how many pounds of each of the metals?

4. Type metal is 77.5% lead, 6.5% tin and 16% antimony. How many pounds of type metal can be made from 64 lb. of antimony? How many pounds of lead and of tin must be used?

5. Type is also made by melting 5 lb. of tin, 9 lb. of antimony, 35 lb. of lead and 1 lb. of copper. Each metal is what per cent. of the alloy?

6. A 2-in. pulley is belted to a 20-in. pulley on a countershaft. The counter contains a 15-tooth gear that meshes with a 90-tooth gear. A revolution of the 90-tooth gear makes one impression of the press. If the driving pulley is making 500 R.P.M., how many impressions does the press make? 1000 R.P.M.? 800 R.P.M.? 1250 R.P.M? 850 R.P.M.?

7. In order that the number of impressions of the press for any given time may be the same, what change would have to be made in the driven pulley if a 3-in. pulley is substituted for the 2-in. pulley?

8. A label for a can 6 in. high and 6 in. in diameter is required. The label is to extend three-fourths of the distance

around the can and within 1 in. of the top and $\frac{3}{4}$ in. of the bottom. How many sheets 25 in. $\times$ 38 in. will be required for 6000 of these labels allowing 8% for waste?

9. The page of a certain magazine contains three columns each 18 picas wide; between each column is 1 pica. The margin on the right-hand side of the page is $5\frac{1}{2}$ picas and there is an equal margin at the left. Find the width of the page in inches.

10. The printed matter of the page of problem 9 is set solid and has 9 lines per in. What size type is used? How many 8-point ems are there per inch?

LESSON LXXIX

FRACTIONAL REVIEW

1. Simplify $\dfrac{5\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}$

2. Simplify $\dfrac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$

3. $13\frac{1}{6} \times 12\frac{1}{13} \times 5\frac{1}{4} \times 6\frac{1}{3} = ?$ $8\frac{1}{9} \times 7\frac{1}{5} \times 3\frac{3}{4} = ?$
$15\frac{1}{3} \times 7\frac{3}{4} \times 8\frac{1}{6} \times 25 = ?$ $187\frac{7}{8} \times 9\frac{3}{7} \times 3\frac{1}{7} = ?$

4. Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$

5. Simplify $\dfrac{\frac{1}{2} + \frac{3}{13} \times \frac{1}{6} - \frac{3}{4} \times \frac{4}{7}}{1\frac{1}{5} \times 10\frac{1}{13} + \frac{3}{4} \times \frac{4}{7}}$

6. What is the value of $\dfrac{3\frac{2}{17} \text{ of } 8\frac{1}{6}}{4\frac{2}{13} \text{ of } 2\frac{1}{16}}$

7. Simplify $\dfrac{2 - \frac{1}{2} \text{ of } (\frac{3}{4} - \frac{1}{20})}{5 + .5 \text{ of } (1 - .9)}$

8. Simplify $\left(\dfrac{522}{6} - \dfrac{2727}{2} + \dfrac{144}{120} + \dfrac{8 \times 9}{4 \times 5}\right) \times 12$

9. $4\frac{5}{18} + 2 \times 5\frac{5}{9} - 3 \times \frac{3}{4} \div \frac{1}{2} = ?$

10. $\frac{7}{8} + \frac{7}{12} + \frac{7}{16} + \frac{7}{20} - \frac{5}{6} - \frac{5}{10} - \frac{5}{14} = ?$

11. Find the sum of $\frac{1}{3} + \frac{1}{6} + \frac{3}{20} + \frac{4}{30}$ correct to four decimal places.

12. $12\frac{1}{2} \times 4\frac{1}{6} \times 5\frac{7}{12} = ?$ $1748 \div (6\frac{1}{3} \times 7\frac{5}{12}) = ?$
$186\frac{6}{12} \times 15\frac{7}{12} \times 1\frac{1}{12} = ?$ $5\frac{5}{12} \times 7\frac{1}{12} \times 18\frac{3}{4} = ?$
$205\frac{5}{12} \times 18\frac{1}{3} \times 5\frac{1}{2} = ?$ $1890 \div (7\frac{1}{3} \times 10\frac{1}{4}) = ?$
$1185\frac{5}{12} \times 9\frac{3}{4} \times \frac{1}{6} = ?$ $1\frac{1}{12} \times 5\frac{1}{3} \times \frac{5}{6} = ?$